Truthmaking, grounding, and Fitch's paradox

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1 Jago (2020) and Loss (forthcoming) have recently used variations on Fitch's paradox to argue that every truth has a truthmaker, and that every fact is grounded. In this paper, I show that Fitch's paradox can also be adapted to prove the exact opposite conclusions: no truth has a truthmaker, and no fact is grounded. All of these arguments are as dialectically effective as each other, and so they are all in bad company.

2 In this section, I will present a slightly unfamiliar formulation of Fitch's paradox, due to Loss (forthcoming: §2). It starts with two schematic premisses:

(FK) $\Box(\mathbb{K}A \rightarrow A)$

(PK) $(A \wedge B) \rightarrow \Diamond(\mathbb{K}A \wedge \mathbb{K}B)$

$\mathbb{K}A$ is meant to be read as *it is known that* A; for the purposes of Fitch's paradox, the $\Diamond$ and the $\Box$ can express any normal modality, but following Jago (2020: 40) and Loss (forthcoming: §2), I will read them as expressing *logical* possibility and necessity, respectively. On this interpretation, (FK) tells us that it is logically impossible to know something false, and (PK) tells us that if a conjunction is true, then it is logically possible that both conjuncts are known.

Together, (FK) and (PK) imply that all truths are known:

(AK) $A \rightarrow \mathbb{K}A$

Here is a proof:

1	$\mathbb{K}\neg\mathbb{K}A \to \neg\mathbb{K}A$	Assumption
2	$\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A$	Assumption
3	$\mathbb{K}A \wedge \neg\mathbb{K}A$	Classical Logic, 1, 2
4	$\neg(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A)$	*Reductio*, 2—3
5	$(\mathbb{K}\neg\mathbb{K}A \to \neg\mathbb{K}A) \to \neg(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A)$	Conditional Proof, 1—4
6	$\Box((\mathbb{K}\neg\mathbb{K}A \to \neg\mathbb{K}A) \to \neg(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A))$	Necessitation, 5
7	$\Box(\mathbb{K}\neg\mathbb{K}A \to \neg\mathbb{K}A) \to \Box\neg(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A)$	Distribution, 6
8	$\Box(\mathbb{K}\neg\mathbb{K}A \to \neg\mathbb{K}A)$	(FK)
9	$\Box\neg(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A)$	Modus Ponens, 7, 8
10	$\neg\Diamond(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A)$	Modal Conversion, 9
11	$(A \wedge \neg\mathbb{K}A) \to \Diamond(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A)$	(PK)
12	$\neg(A \wedge \neg\mathbb{K}A)$	Modus Tollens, 11, 10
13	$A \to \mathbb{K}A$	Classical Logic, 12

3 According to Jago (2020: 43), Fitch's paradox is not a 'genuine paradox, merely a surprising piece of reasoning'. He (2020: 42–3) aims to exploit this kind of reasoning to argue for *truthmaker maximalism*, the doctrine that every truth has a truthmaker:

(AT) $A \to \mathbb{T}A$

where $\mathbb{T}A$ expresses *something makes the proposition* $\langle A\rangle$ *true.* (Following Jago (2020: 40), we can understand x *makes* $\langle A\rangle$ *true* as: $\langle A\rangle$ *is true in virtue of the existence of* x.) Jago argues for (AT) from three premisses:

(FT) $\Box(\mathbb{T}A \to A)$

(DT) $\Box(\mathbb{T}(A \wedge B) \to (\mathbb{T}A \wedge \mathbb{T}B))$

(PT*) $A \to \Diamond\mathbb{T}A$

It is easy to see that (DT) and (PT*) imply:

(PT) $(A \wedge B) \rightarrow \Diamond(\mathbb{T}A \wedge \mathbb{T}B)$

So all we need to do to prove (AT) is take the proof of Fitch's paradox from §2, and swap every $\mathbb{K}$ for a $\mathbb{T}$.

An argument is only as good as its premisses. (FT) and (DT) are uncontroversial truthmaking principles, so the whole weight of the argument lies on (PT*). But on the face of it, (PT*) appears to be a remarkably minimal assumption: all (PT*) tells us is that, for any truth $\langle A \rangle$, it is *logically possible* that $\langle A \rangle$ has a truthmaker. It is important to emphasize that this is just a claim about logical possibility, not metaphysical possibility. You might not think that it is metaphysically possible for any existing thing to make ⟨unicorns do not exist⟩ true. However, it is surely still logically possible, i.e. *logically consistent*, for this truth to have a truthmaker; it is not contradictory to suggest, for example, that it is made true by the *negative fact* that unicorns do not exist.

So, (PT*) appears to be quite a weak assumption. But what positive reason do we have to believe it? Jago (2020: 40–2) offers an inductive argument. He starts with a sample of various uncontroversial truths: ⟨Obama exists⟩, ⟨wombats are marsupials⟩, $\langle 1 + 1 = 2 \rangle$, ⟨scarlet things are red⟩, ⟨unicorns do not exist⟩. He then makes the plausible claim that it is logically possible for each of these truths to have a truthmaker. So, reasoning inductively, Jago (2020: 42) concludes that we have 'warrant — not a proof, but reason nonetheless — to accept' (PT*).

4 Loss (forthcoming) also wants to exploit Fitch's paradox, this time to prove that all facts are grounded:

(AG) $A \rightarrow \mathbb{G}A$

where $\mathbb{G}A$ expresses *it is a grounded fact that A.* He argues for (AG) from these two premisses:

(FG) $\Box(\mathbb{G}A \rightarrow A)$

(PG) $(A \wedge B) \rightarrow \Diamond(\mathbb{G}A \wedge \mathbb{G}B)$

Again, his argument is just Fitch's paradox, but this time swapping the $\mathbb{K}$s for $\mathbb{G}$s. And again, (FG) is an uncontroversial grounding principle, so the weight of the argument lies on (PG). Loss (forthcoming: §3) offers the same kind of inductive argument for (PG) as Jago gave for (PT*). Choose two of your favourite, least controversial candidates for ungrounded facts, A and B. (Loss's preferred candidates are facts of fundamental physics.) We might have excellent reason to believe that A and B are ungrounded. We might even have excellent reason to believe that it is metaphysically impossible for A or B to be grounded. (The one implies the other, if we assume that ungrounded facts metaphysically cannot be grounded.) But it is surely still *logically* possible for A and B to be grounded. So, reasoning inductively, Loss concludes that we have at least some warrant for (PG).

5 I am unconvinced by Jago and Loss's arguments. I want to respond to them in exactly the same way that I want to respond to Fitch's original paradox. What that paradox shows is that (PK) is not the innocent premiss it might initially appear to be: (PK) and (FK) imply (AK), which is absurd; (FK) is entirely uncontroversial; so (PK) must be false. In fact, Fitch's paradox actually provides us with a method for constructing counter-examples to (PK). Take any unknown truth, A. To say that A is an unknown truth is to say that $A \wedge \neg\mathbb{K}A$. As line 10 in Fitch's paradox shows, $\neg\Diamond(\mathbb{K}A \wedge \mathbb{K}\neg\mathbb{K}A)$. So here we have a counter-example to (PK).

As I said, I want to give the same response to Jago and Loss's arguments. What they show is that (PT) and (PG) are not the innocent premisses they appear to be. I do not believe that every truth has a truthmaker, or that every fact is grounded. So I infer that (PT) and (PG) are both false. Indeed, the Fitch-style reasoning actually provides me with a method for constructing counter-examples to these premisses: since I think there are truths without truthmakers, I think that there is an A such that $A \wedge \neg \mathbb{T}A$, even though $\neg\Diamond(\mathbb{T}A \wedge \mathbb{T}\neg\mathbb{T}A)$; and since I think there are ungrounded facts, I think there is an A such that $A \wedge \neg\mathbb{G}A$, even though $\neg\Diamond(\mathbb{G}A \wedge \mathbb{G}\neg\mathbb{G}A)$.

Jago (2020: 43) and Loss (forthcoming: §4) pre-empt this kind of response, and dismiss it as question-begging. Here is how Jago puts the point (focussing on (PT*) rather than (PT)):

> This move is dialectically ineffective, however, for it assumes the falsity of maximalism in taking $A \wedge \neg\mathbb{T}A$ as an example truth. Whether there is such a truthmakerless truth A is precisely what is in question. An effective counterexample to [(PT*)] requires an uncontested truth, for which there is no logically possible truthmaker. But such truths, I suggest, are hard to come by. (Jago 2020: 43)

Maybe Jago and Loss are right that my reply begs the question against them. But if so, then they must likewise beg the question against me, as I will now explain.

6 I want to present two more variations on Fitch's paradox. The first is a bit like Jago's, except rather than proving that *every* truth has a truthmaker, it proves that *no* truth has a truthmaker. Here are its premisses:

(FL) $\Box(\mathbb{L}A \to A)$

(PL) $(A \wedge B) \to \Diamond(\mathbb{L}A \wedge \mathbb{L}B)$

where $\mathbb{L}A$ abbreviates $A \wedge \neg\mathbb{T}A$. (So you can read $\mathbb{L}A$ as: $\langle A\rangle$ *is a truthmakerless truth.*) By the now familiar Fitch-reasoning, we can infer:

(AL) $A \to \mathbb{L}A$

(FL) is entirely unproblematic — in fact it follows trivially from the definition of $\mathbb{L}A$ — and so the argument turns on (PL). But on the face of it, (PL) appears to be a remarkably minimal assumption: all it tells us is that if $\langle A\rangle$ and $\langle B\rangle$ are both true, then it is *logically possible* that they are both truthmakerless truths. Again, it is important to emphasize that this is just a claim about logical possibility, not metaphysical possibility. You might not think that it is metaphysically possible for ⟨Obama exists⟩ to be true without being made true by Obama. However, it is surely still *logically* possible for ⟨Obama exists⟩ to be a truthmakerless truth. Imagine a radically heterodox philosopher who believes that being depends on truth; they would say that Obama exists in virtue of the truth of ⟨Obama exists⟩, not the other way around. Or less radically, imagine a philosopher who denies that there is enough of a gap between Obama's existence and the truth of ⟨Obama exists⟩ for either to hold in virtue of the other (see Trueman forthcoming: ch. 14). These heterodox approaches to truth may strike you as metaphysically absurd, but they are not *logically contradictory*.

As far as I can tell, then, I can offer exactly the same kind of inductive support for (PL) as Jago gave for (PT*). Consider the same sample of uncontroversial truths we considered in §3: ⟨Obama exists⟩, ⟨wombats are marsupials⟩... For any pair of these truths, it is logically possible that they are both truthmakerless truths. So, reasoning inductively, we have at least some warrant for (PL).

My second variation on Fitch's paradox is a bit like Loss's, except rather than proving that *every* fact is grounded, it proves that *no* fact is grounded. Here are its premisses:

(FU) $\Box(\mathbb{U}A \rightarrow A)$

(PU) $(A \wedge B) \rightarrow \Diamond(\mathbb{U}A \wedge \mathbb{U}B)$

where $\mathbb{U}A$ abbreviates $A \wedge \neg\mathbb{G}A$. (So you can read $\mathbb{U}A$ as: *it is an ungrounded fact that A*.) Yet again, we can infer:

(AU) $A \rightarrow \mathbb{U}A$

(FU) is trivial, and so the argument turns on (PU). But as far as I can tell, I can offer exactly the same kind of inductive support for (PU) as Loss gave for (PG). Choose two of your favourite, least controversial candidates for grounded facts, A

and B. (Maybe A is a fact about what is funny, and B is a fact about what is polite.) We might have excellent reason to believe that A and B are grounded. We might even have excellent reason to believe that it is metaphysically impossible for A or B to be ungrounded. (The one implies the other, if we assume that grounded facts metaphysically cannot be ungrounded.) But it is surely still *logically* possible for A and B to be ungrounded. So, reasoning inductively, we have at least some warrant for (PU).

7 To be clear, I do not expect Jago or Loss to be convinced by my arguments, any more than I was convinced by theirs. I would expect them to reject (PL) and (PU), just as I rejected (PT) and (PG). Indeed, by their lights, they have a method for constructing counter-examples to (PL) and (PU): since Jago thinks some truths have truthmakers, he thinks that there is an A such that $A \wedge \neg \mathbb{L}A$, even though $\neg\Diamond(\mathbb{L}A \wedge \mathbb{L}\neg\mathbb{L}A)$; and since Loss thinks some facts are grounded, he thinks there is an A such that $A \wedge \neg\mathbb{U}A$, even though $\neg\Diamond(\mathbb{U}A \wedge \mathbb{U}\neg\mathbb{U}A)$.

However, these replies to my arguments are no better than my replies to Jago and Loss. If my replies begged the question against Jago and Loss, then these replies beg the question against me too: they presuppose that some truth has a truthmaker, and that some fact is grounded, which is precisely what is in question. What Jago and Loss really need are counterexamples to (PL) and (PU) with uncontroversial antecedents, but those will be hard to come by.

What we are left with, then, is a kind of *bad company* objection. None of the arguments we have reviewed in this paper is any more dialectically effective than any of the others. So since these arguments pull in different directions, none of them is dialectically effective, *full stop*.[1]

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References

Jago M. 2020. A short argument for truthmaker maximalism. *Analysis* 80: 40–44.

Loss R. forthcoming. There are no fundamental facts. *Analysis* (available via Advance Articles).

Trueman R. forthcoming. *Properties and Propositions: The Metaphysics of Higher-Order Logic*. Cambridge: Cambridge University Press.