The Foundations of Modality: From Propositions to Possible Worlds, by Peter Fritz. Oxford: Oxford University Press, 2023. Pp. xi + 203.

This is a book that is exactly described by its title. Fritz begins by presenting a higher-order theory of propositions, i.e. a theory which identifies propositions with the values of variables that appear in sentence-position. He then uses that theory to argue that there is a broadest notion of necessity. And he ends the book by arguing that certain privileged propositions play the role of possible worlds.

The Foundations of Modality (*FM* for short) is beautifully clear. Fritz has an incredible knack for presenting complex formal material in an accessible way. (From now on, everyone should learn about the Russell-Myhill Paradox from Chapter 3.) Fritz is also an extremely good judge of which formal demonstrations to include in the main text of a chapter, and which to save for supplementary appendices. As a result, *FM* provides an excellent way into some of the most important recent developments in higher-order logic: even if you never dip into the supplementary material, you can still come away with substantial knowledge of the established results, and in many cases of exactly how they are established. Moreover, Fritz adds valuable new ideas and insights at every step along the way. All of this makes *FM* essential reading for anyone interested in higher-order or modal metaphysics.

The rest of this review is structured as follows. In §1, I will give a slightly more detailed account of Fritz's argument. Then, in §§2 & 3, I will offer a critical discussion of two aspects of that argument. However, I want to emphasise now that these criticisms are not meant to detract from my positive comments about the book. They merely reflect the fact that a philosopher can always find something to disagree with.

1 Overview

FM is divided into five parts (plus an introduction), and each part consists of two chapters. Part 1 introduces the higher-order language that Fritz uses throughout the book. In Chapter 1, Fritz motivates his decision to use higher-order resources, which permit us to bind variables outside of name-position. Russell's Paradox shows that naïve theorising about properties is inconsistent, and Fritz recommends going higher-order as the best way of restoring consistency. This is a popular way of motivating higher-order logic, but for what it is worth, I have my doubts. Higher-order logic blocks Russell's Paradox by forbidding you from even asking whether a property does or does not apply to itself; but absent an independent argument for this expressive limitation, this looks dangerously like 'solving' Russell's Paradox just by pretending not to understand it. (I try to give the missing independent argument in Trueman 2021. Also see *FM*: 167, where Fritz offers a response to the kind of worry I have raised here.) But whether or not you are convinced by Fritz's case for higher-order logic, there is much to learn from how he goes on to use it.

In Chapter 2, Fritz gives a formal presentation of his higher-order language, $\mathcal{L}$. Fritz makes the extremely helpful decision to limit the resources of this language to just the ones he will need, which allows him to simplify a number of the proofs he gives later in the book. $\mathcal{L}$ has two types of variable. The first are propositional variables, which appear in sentence position, and are usually written as p or q. They are called 'propositional variables' because, as a matter of stipulation, their values are called 'propositions'. The other variables appear in the position of sentential operators, and are usually written as m or n. Intuitively, their values are relations between propositions, which Fritz calls 'modalities'. These two types of variable are designed to work together: every propositional variable is a formula, and $m(\phi_1, \ldots, \phi_n)$ is a formula whenever each ϕ_i is a formula and m is an n-adic sentential operator. $\mathcal{L}$ also includes the usual stock of logical constants, including quantifiers, a λ-operator and the identity-symbol: both types of variable can be bound by the quantifiers, but only (finite sequences of) propositional variables can be bound by the λ-operator, and only propositional variables can flank the identity-symbol. Fritz presents a proof system for $\mathcal{L}$, $\vdash^=$, which behaves exactly as you would expect.

In Part 2, Fritz argues for a coarse-grained individuation of propositions. He begins in Chapter 3 by presenting some limitative results on how finely propositions can be individuated in $\vdash^=$, chiefly the Russell-Myhill Paradox. The literature on this paradox is now fairly vast, and it provides all sorts of ways of tweaking $\vdash^=$ to dodge the result. However, they all come with their own problems, and Fritz is surely right that it is worth exploring what happens when we individuate propositions coarsely. In Chapter 4, Fritz presents his preferred coarse-grained theory of propositions, *Classicism*. This theory takes $\vdash^=$, and closes it under the *Rule of Equivalence*:

(RE) If Classicism $\vdash^= \phi \leftrightarrow \psi$, then Classicism $\vdash^= \phi = \psi$

A noteworthy consequence of Classicism is that there is just one tautologous proposition, $\top$, and just one contradictory proposition, $\bot$.

In Part 3, Fritz uses Classicism to develop a theory of metaphysical necessity. In Chapter 5, he proposes defining metaphysical necessity as *being identical to* $\top$, which can be formalised as $\lambda p.p = \top$, and which Fritz abbreviates as $\Box$. He motivates this definition by arguing that, given some key assumptions, $\Box$ is the *broadest* objective necessity: it is necessary, in every objective sense of 'necessary', that if $\Box\phi$ then ϕ is necessary in every objective sense of 'necessary'. (The 'objective' qualification is meant to filter-out *representational* necessities. Following Fritz's lead, I will leave that qualification implicit from now on.) As I will explain in §2, I think that one of Fritz's initial assumptions is problematic, but happily, we can revise that assumption and still establish that $\Box$ is the broadest necessity.

At the end of Chapter 5, Fritz shows that Classicism entails that $\Box$ obeys the modal logic S4. However, as Fritz explains at the beginning of Chapter 6, Classicism does not imply the S5 axiom for $\Box$:

(S5) $\Diamond\phi \to \Box\Diamond\phi$

Fritz argues for (S5) by extending $\mathcal{L}$ with an actuality operator, and showing that (S5) follows if the actuality operator satisfies some standard principles. I have to admit that I found this argument a little bit slippery, since, as he warns on p.112, Fritz extends $\mathcal{L}$ in Chapter 8 with new resources that are not easily combined with an actuality operator. For my money, I would have been happy simply to accept (S5) as another assumption, or better yet, the *Necessity of Distinctness*,

(ND) $\phi \neq \psi \to \Box(\phi \neq \psi)$

which is equivalent to (S5) given Classicism.

Part 4 is the culmination of the book. Let a proposition, p, be *maximal* iff for every proposition q, p strictly entails q or p strictly entails $\neg q$, but not both. Formally:

$$Mp := \forall q(\Box(p \to q) \leftrightarrow \neg\Box(p \to \neg q))$$

In Chapter 7, Fritz demonstrates that maximal propositions play the theoretical role of possible worlds iff the following *Atomicity* principle is true:

(A) $\Box\exists p(Mp \wedge p)$

Unfortunately, Classicism (even when supplemented with (S5)) does not imply (A). In Chapter 8, Fritz recommends that we overcome this limitation by extending $\mathcal{L}$ with *plural propositional variables*, which are to propositional variables as standard plural variables are to first-order variables. Fritz shows that, if these plural propositional variables obey straightforward analogues of standard plural principles, then we can use them to prove (A). However, as I will argue in §3, there is reason to doubt the intelligibility of Fritz's plural propositional variables, and so I would rather justify (A) in some other way.

Part 5 is an epilogue. In Chapter 9, Fritz gives an overview of his coarse-grained world, and in Chapter 10, he identifies various points where an opponent might resist his argument. This struck me as a remarkably magnanimous and undogmatic way to end the book.

2 Contingent necessities

Recall that Fritz's aim in Chapter 5 is to argue that $\Box$, i.e. $\lambda p.p = \top$, is the broadest necessity. His argument relies upon Classicism, plus two further assumptions (*FM*: 94):

(1) $\text{Nec}(\Box)$
(2) $\text{Nec}(n) \to \Box n\top$

where $\text{Nec}(n)$ says that n is a necessity. Fritz formalises the claim that one modality is at least as broad as another as follows:

(3) $m \sqsubseteq n := \forall p \Box (mp \rightarrow np)$

It is then easy to show that $\Box$ is at least as broad as every necessity:

(4) $\mathrm{Nec}(n) \rightarrow \Box \sqsubseteq n$

Classicism implies the following chain of identities: $\forall p \Box(\Box p \rightarrow np) = \forall p \Box(p = \top \rightarrow np) = \Box n\top$. Now suppose $\mathrm{Nec}(n)$. (2) implies $\Box n\top$, i.e. the last link in our chain of identities, and so we can infer $\Box \sqsubseteq n$, i.e. the first link in our chain. (4) follows by conditional proof. (Also note that there was no sleight of hand involved in defining $\sqsubseteq$ in terms of $\Box$: since $\Box$ is factive, (4) implies all of its own substitution instances that result from swapping the $\Box$ in (3) for any other necessity.)

Fritz's reasoning is straightforward, and so everything hangs on his assumptions. Fritz (*FM*: 93) motivates (1) simply by pointing out that $\Box$ behaves exactly like a necessity; in particular, Classicism implies that it obeys the modal logic S4 (*FM*: 98–9). His argument for (2) is a bit more delicate. He starts with the intuitive thought that, if n is a necessity, then it should be true that $n\top$. This thought is fine so far as it goes, but, Fritz insists, we really need something stronger. It should not just be a contingent accident that $n\top$. For example, consider the modality *has been asserted by Rob Trueman*. As it happens, I have asserted $\top$. (I made sure to shortly before writing this paragraph.) But it is still possible, in all sorts of senses of 'possible', that I never asserted $\top$. Fritz takes this to be a good reason to deny that *has been asserted by Rob Trueman* is a necessity. More generally, Fritz claims that if n is a necessity, then $n\top$ should be necessary in every sense. So, since we have already accepted assumption (1), we should also accept assumption (2). (Bacon (2018: 735, 2024: 141) offers a similar argument.)

I am not convinced by Fritz's case for (2). Let n be *physical necessity*. Clearly, $\mathrm{Nec}(n)$ is actually true. But now imagine a chaotic world governed by no physical laws. (I assume that it is possible for there to be no physical laws, at least in the sense that the proposition that there are no laws is not identical to $\bot$. See Roberts (2022: 1233–5 & 1238–40) for a defence of this assumption.) What does n apply to in a world like that? It is not implausible to say that it applies to *nothing*. After all, for p to be physically necessary is for it to be (in some sense) necessitated by the laws of physics. And if there are no laws of physics, then nothing is necessitated by them, not even $\top$. (This is the verdict given by Hale and Leech's (2017) account of physical necessity.) I agree that this would imply that, in a lawless world, n would not be a necessity. But I do not see why it should make me doubt that n is a necessity in our well-behaved, law-governed world. So, I would be happy to maintain that $\mathrm{Nec}(n)$ is actually true, despite the fact that $\neg\Box n\top$.

Now, I should admit that this counterexample to (2) is not watertight. It is not exactly clear what to say about physical necessity in a lawless world. (Roberts (2020: §4), for example, offers an alternative account which implies that, in a lawless world, $\top$ is still physically necessary; however, the main motivation for his account is that it preserves $\Box$ as the broadest necessity, which would be question-begging in this context.) But whatever the right account of physical necessity turns out to be, I hope that the foregoing at least makes it clear that (2) is a substantive premise. It is an appealing premise if you assume that every necessity is necessarily a necessity, but not if you don't.

You might be tempted to reply that Fritz is really just assuming that every necessity satisfies the standard modal principle of *Necessitation*. (See Roberts 2020: 714, 2022: 1227–8.) Here is how you would need to be thinking of Necessitation to give this reply:

(N) If Classicism $\vdash^{=} \phi$, then Classicism $\vdash^{=} n\phi$

Clearly, Classicism $\vdash^{=} \top$, and so (N) implies that Classicism $\vdash^{=} n\top$, and thus Classicism $\vdash^{=} n\top \leftrightarrow \top$. But Classicism is closed under (RE), and so Classicism $\vdash^{=} n\top = \top$, i.e. $\Box n\top$.

The first thing that makes this line of reasoning tricky is that whether a modality satisfies (N) depends in part on how we express it. *No* modality satisfies (N) when we express it with a primitive operator. But setting that aside, it is still too much to assume that every necessity can be expressed in a way that satisfies (N). Classicism does not keep a complete record of which modalities are necessities. There is no reason to expect that a necessity picked at random will be expressed by some operator, $\blacksquare$, such that Classicism recognises $\blacksquare$ as a necessity and so implies $\blacksquare\top$.

The closest we can reasonably expect to get to (N) is this:

(N*) If Classicism+Nec$(n) \vdash^{=} \phi$, then Classicism+Nec$(n) \vdash^{=} n\phi$

(If we even get this close will depend on how we define Nec.) (N*) obviously provides no guarantee for (2), since we have no reason to think that every consequence of Classicism+Nec(n) is identical to $\top$. But (N*) does provide an explanation for why *has been asserted by Rob Trueman* is not a necessity. If n is a necessity, then (N*) implies that $n\top$, and that $nn\top$, and that $nnn\top$, and that... I have only ever asserted finitely many things, and so while I have asserted $\top$, I have not also asserted that I asserted $\top$, and that I asserted that I asserted $\top$, and... Of course, we could idealise my finite limitations away, and suppose that I had asserted all of those things. But, personally, I find that my intuitions dry up under this idealisation. If I really did make all of these assertions, then maybe *has been asserted by Rob Trueman* really would be a necessity.

Fortunately, however, we can still prove that $\Box$ is the broadest necessity without assuming that every necessity is necessarily a necessity. We begin by tweaking (1) and (2):

(1*) $\Box$Nec$(\Box)$
(2*) $\Box($Nec$(n) \rightarrow n\top)$

The motivation for (1*) is very similar to the motivation for (1): since Classicism itself implies that $\Box$ behaves like a necessity, it seems plausible to identify Nec$(\Box)$ with $\top$. We could motivate (2*) by offering Nec$(n) \rightarrow n\top$ as a conceptual truth, and then suggesting that, in general, every conceptual truth is identical to $\top$. Alternatively, we could take a more metaphysical approach, and propose that *part of what it is* to be a necessity is to apply to $\top$. We can formalise this in $\mathcal{L}$ as follows:

(5) $\forall n \exists p(\mathrm{Nec}(n) = (n\top \wedge p))$

Given Classicism, (5) implies that $\exists p((\mathrm{Nec}(n) \rightarrow n\top) = ((n\top \wedge p) \rightarrow n\top))$, which in turn implies that $(\mathrm{Nec}(n) \rightarrow n\top) = \top$, i.e. (2*).

These new premises will not allow us to prove (4) if we stick to (3) as our definition of broadness. (If we do not assume that $n\top = \top$, then we have no way of showing that $(p = \top \rightarrow n\top) = \top$.) However, we can prove (4) if we also tweak (3):

(3*) $m \sqsubseteq n := \Box(\mathrm{Nec}(n) \rightarrow (\mathrm{Nec}(m) \wedge \forall p(mp \rightarrow np)))$

Intuitively, this new definition stipulates that m is at least as broad a necessity as n just in case: it is impossible for n to be a necessity without agreeing that everything certified by m as necessary is indeed necessary. It seems to me that this is exactly the right definition of broadness to give, if we do not assume that every necessity is necessarily a necessity.

From (1*)–(3*), Classicism implies the following chain of identities: $(\mathrm{Nec}(n) \rightarrow (\mathrm{Nec}(\Box) \wedge \forall p(\Box p \rightarrow np))) = (\mathrm{Nec}(n) \rightarrow \forall p(p = \top \rightarrow np)) = (\mathrm{Nec}(n) \rightarrow n\top) = \top$. (4) follows immediately. Moreover, as far as I can tell, making the changes I have suggested here would have no adverse effects on the rest of Fritz's arguments in *FM*. So I offer it as a minor revision that will allow him to accommodate contingent necessities.

3 Against plural propositional quantification

In Chapter 7 of *FM*, Fritz shows that maximal propositions play the theoretical role of possible worlds iff the Atomicity principle (A) is true. Unfortunately, however, at that point in the book, Fritz does not have the resources to show that (A) is true. So, in Chapter 8, he gives himself some more. He introduces new *plural propositional variables*, written as $pp, qq, \ldots$ Fritz lays down some formal principles governing these variables, and then uses them to prove (A).

Plural propositional variables are a novelty, and so Fritz makes three stipulations to narrow down their interpretation (*FM*: 144):

(i) They must obey the formal principles that Fritz has laid down
(ii) Quantifiers binding them must roughly correspond to certain plural constructions in natural language, such as 'Some propositions' in 'Some propositions could each be true, but could not all be true together'
(iii) They must relate to propositional variables as standard plural variables relate to first-order variables

Stipulation (i) is straightforward, but as Fritz (*FM*: 162) acknowledges, his formal principles leave a lot of interpretative leeway. (I will return to this point later.) Stipulation (ii) is suggestive, but it is limited by the fact that any correspondence really will have to be *rough*. The problem is that 'Some propositions' is a *nominal* quantifier: it binds pronouns which appear in name-position. Fritz (*FM*: 143) is explicit that plural propositional variables are meant to be plural versions of standard propositional variables, and they appear in sentence-position, not name-position. This leaves stipulation (iii). Fritz does not tell us much about the analogy he intends to draw between first-order and propositional plurals, but here is my attempt to fill in the details.

We start with the first-order case. For simplicity, we will focus on constants rather than variables, but nothing substantive hangs on this. A first-order singular constant is a name that refers to exactly one thing: 'Peter Fritz' refers to, and only to, Peter Fritz. A first-order plural constant is *also* a name, but one which happens to refer to more than one thing: 'Peter Fritz and Rob Trueman' refers to Peter Fritz and Rob Trueman, and nothing else.

What about the propositional case? A singular propositional constant is a sentence that expresses exactly one proposition: 'Grass is green' says that grass is green, and it says nothing else. So, (iii) seems to require that a plural propositional constant *also* be a sentence, but one which happens to express more than one proposition: perhaps '*pp*' says that grass is green, *and* says that grass is purple. Now, in Fritz's official syntax (*FM*: 144), plural propositional terms belong to a different type from sentences. But, given this reading of (iii), that type-distinction ought to be collapsed. Sentences in general are terms that express propositions. A singular propositional term is a sentence that happens to express just one proposition, and a plural propositional term is a sentence that happens to express many propositions.

Of course, in a sense, we are all familiar with the idea that a single sentence might express more than one proposition. That is the standard way of describing what an ambiguous sentence does. But it is important to emphasise that plural propositional constants are not meant to be ambiguous sentences. Even when fully disambiguated, and nailed down to just one interpretation, they *still* express more than one proposition.

However, I doubt that it makes sense for an unambiguous sentence to express more than one proposition. Or at least, I doubt that it makes sense if those propositions have different truth-values, and Fritz needs plural propositional terms like that for his proof of (A). Let's continue to interpret '*pp*' as saying that grass is green, and saying that grass is purple. (It does not matter if '*pp*' also says anything else.) '*pp*' is meant to be a sentence, and the hallmark of a (declarative) sentence is that it is *truth-evaluable*. So which is '*pp*', true or false? It says that grass is green, and grass is green, so it should be true. Equally, though, it also says that grass is purple, and grass is not purple, and so it should be false. But (setting dialetheism aside) it can't be both.

The obvious reply is that '*pp*' should be evaluated conjunctively: '*pp*' is true iff *every* proposition it expresses is true; otherwise, '*pp*' is false. (Importantly, this is not to identify a conjunction with the plurality of its conjuncts. A conjunction is a single proposition, not a plurality.) On this conjunctive evaluation, '*pp*' is false, because it expresses the false proposition that snow is purple. But why should we evaluate '*pp*' *conjunctively* rather than *disjunctively*: '*pp*' is true iff *some* proposition it expresses is true; otherwise it is false? Evaluated disjunctively, '*pp*' is true, because it expresses the true proposition that grass is green. There is no obvious reason to prefer the conjunctive evaluation over the disjunctive, or vice versa, and so we seem to be faced with an arbitrary choice. But arbitrary choices like this are often a sign that something has gone wrong.

In fact, things get worse. Suppose we did just choose, arbitrarily, to evaluate '*pp*' conjunctively. So '*pp*' is false. There is (what I take to be) a platitudinous connection between falsehood and negation: '*pp*' is false iff '¬*pp*' is true. But what does the sentence '¬*pp*' say? There should be a compositional clause which determines what '¬*pp*' says in terms of what '*pp*' says. Here is a natural suggestion:

(¬) $\forall q$('*pp*' says that $q \rightarrow$ '¬*pp*' says that $\neg q$)

So according to (¬), '¬*pp*' is itself a plural propositional term, which says that grass is not green, and that grass is not purple (and maybe some other things too). So, when we apply the conjunctive evaluation to '¬*pp*', we get the result that it is false — because it expresses the false proposition that grass is not green — rather than true. (I presented a similar argument in Trueman 2021: 43–5.)

In effect, if we start with a *conjunctive* evaluation of '*pp*', the rule (¬) forces us to treat '¬*pp*' as if it were the negation of a *disjunction*. Similarly, if we started with the disjunctive evaluation of '*pp*', (¬) would force us to treat '¬*pp*' as if it were the negation of a conjunction: '¬*pp*' would be true — because it expresses the true proposition that grass is not purple — rather than false. So, in the presence of (¬), neither evaluation is stable.

Of course, you could give up on (¬). Here are two alternatives you might try:

($\neg^{\wedge}$) '¬*pp*' says that $\exists q$(('*pp*' says that q) $\wedge \neg q$)
($\neg^{\vee}$) '¬*pp*' says that $\forall q$(('*pp*' says that q) $\rightarrow \neg q$)

The conjunctive evaluation of '*pp*' can stably be combined with ($\neg^{\wedge}$), and the disjunctive evaluation can stably be combined with ($\neg^{\vee}$). However, both rules are problematic in another way: they make negation *meta-linguistic*. But '¬*pp*' belongs to the object-language along with '*pp*'. Negation is not a tool for linguistic ascent, but, in Wittgenstein's (1922: 5.2341) memorable phrase, for reversing sense.

There are other replies that you could try in defence of plural propositional terms. However, my hunch is that none of them will work, because there is just a fundamental failure of fit between plurality and propositionality. Propositional terms are truth-evaluable. But a plurality is a mere collection of things. And if there is one lesson to learn from the (admittedly vexed) literature on the unity of the proposition, it is that mere collections cannot be true or false.

Having said all of this, I should repeat that Fritz's official syntax distinguishes between sentences and plural propositional terms. The motivation for dissolving that distinction stems from Fritz's stipulation (iii): since singular propositional terms are sentences, plural propositional terms should be sentences too. Fritz could, then, simply abandon (iii), and reinstate the type-distinction between plural and singular propositional terms.

But if plural propositional terms are *not* sentences that express multiple propositions, then what are they? One attractive suggestion would be that plural propositional terms are monadic operators, like $\neg$ and $\Box$. After all, monadic operators express properties of propositions, and it is well known that properties can serve as collections of their instances. To do the work that Fritz needs of them, plural propositional terms would need to express *rigid* properties, whose extensions cannot change. This would be a new kind of property, not previously introduced in *FM*, but as Fritz (*FM*: 147 & 162) notes, the details of this approach have already been worked out by Gallin (1975: pt. II). And it turns out that Gallin's theory of rigidified properties provides all the resources you need to prove (A).

It is an interesting question why Fritz chose to introduce his plural propositional variables rather than rely on Gallin's rigidified properties. The most he tells us is that 'the best way of justifying [Gallin's theory of rigidified properties] may be by appealing to the intelligibility of plural quantification' (*FM*: 162). This remark obviously leaves a fair amount to the reader. My best guess is that Fritz was attracted by the apparent *innocence* of plural locutions. It is widely believed that plural first-order terms are ontologically innocent: introducing plural first-order terms does not introduce anything new into our ontology; it just allows us to talk about our old ontology in new ways. Likewise for plural propositional terms: they just let us speak about our old propositions in new ways; so, if these new ways of speaking allow us to prove (A) from premises we already accept, then (A) was always a consequence of those premises, even if we lacked the words to show it. By contrast, merely stipulating that there are rigidified properties looks dangerously like just stipulating that there are possible worlds from the outset. However, that motivation obviously leans heavily on the problematic analogy between plural propositional terms and their first-order counterparts.

To end, though, let me repeat that none of the criticisms I have offered are in any way meant to undermine the value of *FM*. Fritz has laid out a path that starts from a coarse-grained theory of propositions, goes via a theory of necessities, and ends in a theory of worlds. He is always clear about what each step on that path involves. The only downside of his being so clear is that, very occasionally, you might disagree with him.*

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