

Properties and Propositions

The Metaphysics of Higher-Order Logic

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For Michael Trueman, who never would have read this book anyway.

Contents

	<i>Acknowledgments</i>	<i>page ix</i>
	<i>Introduction</i>	1
1	Substitution in a Sense	8
	1.1 Some counterexamples to the Reference Principle	10
	1.2 Sense-substitution	11
	1.3 A new Reference Principle	14
	1.4 The sense in sense-substitution	18
	1.5 A circularity objection	20
	1.6 The differences between Reference Principles	21
2	The Term/Predicate Distinction	23
	2.1 The senses of terms and the senses of predicates	23
	2.2 Substituting predicates for predicates	25
	2.3 Substituting predicates for terms	28
	2.4 Substituting terms for predicates	30
	2.5 Type-shifting principles	31
	2.6 An assumed distinction	33
3	Ramsey's Challenge	35
	3.1 Socrates and wisdom	36
	3.2 Complex predicates	38
	3.3 Ramsey's alternative scheme	39
	3.4 Subsential monism	44
	3.5 Back to the term/predicate distinction	46
4	Two Types of Reference	48
	4.1 The argument in brief	49
	4.2 Preliminaries: disquotation and singular terms	51
	4.3 The disquotation problem for predicate-reference	54
	4.4 Disquotation and sense-substitution	55

4.5	A different type of reference	57
4.6	No co-reference between terms and predicates	58
4.7	Looking ahead	59
5	The Reference Principle	61
5.1	Sense-substitution	62
5.2	Premise 0	63
5.3	Premise 1	64
5.4	Premise 2	64
5.5	Premise 3	64
5.6	The proof	67
6	Fregean Realism	69
6.1	Predicate-reference and satisfaction	70
6.2	Properties are satisfaction conditions	74
6.3	The property/object distinction	78
7	Fregean Nominalism	80
7.1	Two different ways of reading (S)?	81
7.2	Against a Quinean suggestion	82
7.3	Non-nominal quantification	84
7.4	Set-theoretic semantics	86
7.5	Quantification over satisfaction conditions	90
7.6	Beyond nominalism and realism	91
7.A	Appendix: Second-order semantics	92
8	Some Alternatives	96
8.1	Wright's ascription	96
8.2	Hale's primary/secondary distinction	98
8.3	MacBride's impure reference	100
8.4	Wiggins, Strawson and the copula	105
9	The Concept <i>Horse</i> Paradox	108
9.1	The property <i>horse</i> is not a property?	109
9.2	Categorial differences	111
9.3	Categorial similarities	114
9.4	Saying and showing	116
10	The Metaphysics Exam	120
10.1	The Problem of Universals	120
10.2	Bradley's Regress	126
10.3	Are properties immanent or transcendent?	133
10.4	Converse relations	137

11	States of Affairs	145
	11.1 Fregean realism about states of affairs	145
	11.2 Negative and disjunctive states of affairs	148
	11.3 Facts as obtaining states of affairs	150
	11.4 The structure of states of affairs	152
12	The Prenective View of Propositional Content	156
	12.1 Propositional content	156
	12.2 Two views of propositional content	157
	12.3 Formalising inferences	159
	12.4 The Prenective View versus natural language	162
	12.5 The Standard View versus natural language	165
13	On Content and Object	168
	13.1 The limits of the Standard View	168
	13.2 A truth conditional approach	171
	13.3 The constituents of propositions	173
	13.4 Propositions are propositional contents	178
	13.A Appendix: Contemporary approaches to structured propositions	181
14	The Identity Theory of Truth	188
	14.1 An immodest identity theory	189
	14.2 The core of the identity theory	191
	14.3 The identity theory as a form of direct realism	193
	14.4 Falsehood and unity	195
	14.5 The individuation problem	197
	14.6 Truths unmade and ungrounded	199
	14.7 Believing the facts	203
	<i>References</i>	207
	<i>Index</i>	219

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¹ <https://creativecommons.org/licenses/by/4.0/>

Introduction

Properties exist, but then again they don't. That is the fundamental thought of this book. Properties do not exist in the same way as ordinary objects, like knives and forks and tables and chairs. They don't even exist in the same way as unusual objects, like the number 9 or our galaxy's centre of mass. They exist in a way all of their own.

It is easy to see why a view like this would be attractive: it promises all the benefits of believing in properties, without any of the drawbacks of actually believing in them. It is also easy to see why it might seem like cheating. In fact, after Quine (1948) it became orthodoxy to insist that there is only *one* kind of existence, and it is expressed by the first-order existential quantifier, $\exists x\dots x\dots$. More recently, however, this orthodoxy has been on the decline. Some philosophers (e.g. Hirsch 2011) have insisted that existential quantifiers in different languages can express different notions of existence, and no one of those notions is privileged over the rest. Other philosophers (e.g. Sider 2011, §9.3) have claimed that in most of its uses, $\exists x\dots x\dots$ does not express existence at all; it only does that when it is used in a special language designed for discussing metaphysics, so-called Ontologese.

I want to be clear right from the start that I am not planning on challenging the Quinean orthodoxy in either of these ways. I am happy to grant that the first-order existential quantifier is univocal, and expresses existence in all of its uses. What I want to challenge is the idea that the notion of existence expressed by $\exists x\dots x\dots$ is the *only* notion of existence. Let me explain.

Properties as second-order existents

Earlier I distinguished between properties and objects. What does this distinction amount to? Throughout this book, I will use 'object' as a catch-all for

anything which can be referred to with a singular term, and ‘property’ as a catch-all for anything which can be referred to with a predicate. Another way of putting the main thesis of this book is: properties are not objects.¹ If we assume for the moment that properties do exist, then it is easy to show that this thesis is equivalent to my original claim that they exist in a different way from objects.

First, we need to draw a distinction between different kinds of quantifier. *First-order* quantifiers bind first-order variables, i.e. variables in term-position. For example, the quantifier in ‘ $\exists x$ x is a philosopher’ binds the variable x , which is in the position of a singular term, like ‘Socrates’ or ‘Plato’. Other kinds of quantifier bind variables in different kinds of position. Most important for our purposes, *second-order* quantifiers bind variables in predicate-position. For example, the quantifier in ‘ $\exists X$ X (Socrates)’ binds the variable X , which is in the position of a predicate, like ‘() was a philosopher’ or ‘() drank hemlock’; very roughly, ‘ $\exists X$ X (Socrates)’ says that Socrates has some property.

The kind of existence which suits objects is the kind that is expressed by first-order existential quantification, $\exists x\dots x\dots$. An object exists just in case it falls within the domain of some first-order quantifier. That is a Quinean doctrine which I have no desire to dispute. But if properties are not objects, then they cannot fall within the domain of a first-order quantifier. An object is anything which can be referred to with a singular term, and everything in the domain of a first-order quantifier can be referred to with a singular term. This is an immediate consequence of the fact that first-order quantifiers bind variables in term-position. When we assign a value to a variable in term-position, we in effect transform that variable into a term referring to that value. For example, when we assign Socrates to the variable x in ‘ x is a philosopher’, that *variable* comes to behave essentially as a *term* referring to Socrates. So if properties are not objects, then they cannot appear in the domain of a first-order quantifier.

But this does not mean that we cannot quantify over properties at all: we just need to use *second-order* quantifiers. Second-order quantifiers bind variables in predicate-position, and properties are the things that predicates can refer to. So just as the members of a first-order domain are all objects, the members of a second-order domain are all properties. The kind of existence which suits properties is thus the kind that is expressed by second-order existential quantification, $\exists X\dots X\dots$. A property exists just in case it falls within the domain of some second-order quantifier. And crucially, the kind of existence expressed by $\exists X\dots X\dots$ is completely distinct from the kind expressed by $\exists x\dots x\dots$. In fact,

¹ As we will see in Chapter 4, this is not quite an accurate formulation of my view. What I really want to say is that it is *nonsense* to suppose that a property might be an object, but we won’t get anywhere if we let ourselves get hung up on that now.

if properties are not objects, then the domains of these two quantifiers cannot even overlap.

So if we assume that properties exist, then the claim that properties are not objects entails that properties and objects exist in two different ways: objects exist in the *first-order* way, and properties exist in a distinct *second-order* way.² And clearly, the reverse entailment holds too: if properties and objects exist in different ways, then no property is an object.

Of course, we do not yet actually have an argument that properties are not objects. Nor do we have an argument for the background assumption that properties exist. Why should we think of predicates as referring expressions in the first place? I will give my arguments in Chapters 1–7. In short, I will argue that the semantic roles of terms and predicates are so different that it just does not make any sense to suppose that they might co-refer (Chapters 1–5); and once we appreciate all that this entails, we will see that there is really nothing to object to in the claim that predicates refer to properties (Chapters 6–7).

The concept *horse* paradox

Everything I have so far said was taken more or less straight from Frege. It was Frege who suggested that we conceive of objects and properties as the referents of terms and predicates, and Frege who insisted that in this sense, no property is an object. The only difference is that Frege used the word ‘concept’ instead of ‘property’, but that was an unhelpful terminological decision: ‘concept’ comes with some unwanted cognitive connotations, which ‘property’ does not. Indeed, I have taken so much from Frege that I will call my position, according to which properties exist but only in their own second-order sense, *Fregean realism*.³

So according to Fregean realism, no property is an object. However, as Frege himself recognised (1892a), this doctrine has a rather unsettling consequence. If properties are not objects, then singular terms cannot refer to them, not even terms that look custom built for the job, like ‘the property *horse*’. In accordance

² There is a point of contact here with the *ontological pluralism* of McDaniel (2009b, 2017) and Turner (2010, 2012). They both argue that there are multiple kinds of existence, and that they are expressed by different kinds of existential quantifier. But crucially, McDaniel and Turner are dealing with different kinds of *first-order* quantifier. In other words, they are concerned with the kinds of existence that objects can enjoy. By contrast, I want to deny that properties are any kind of object at all.

³ To be absolutely clear, the view that I will present in this book is not intended to be an accurate reconstruction of what Frege himself thought about properties. It is just a view that is fundamentally Fregean in the way described above.

with Frege's own terminology, this has become known as the concept *horse* paradox.

At least in his published writings, Frege was rather unfazed by this paradox. He simply dug in his heels, declared that the property *horse* is not a property, and insisted that properly understood, there is nothing paradoxical about that. In the end, I think that Frege was right that there is no real paradox here. But a lot more needs to be said to dissolve the *sense* of paradox. Much of what I have to say in Chapters 6 and 7 will be said with an eye to doing just that, although I will save the full discussion of the paradox for Chapter 9. For now, I will just give a quick sketch of what I will say.

The reason that the concept *horse* paradox feels paradoxical is not just because 'the property *horse*' really looks like it should refer to a property. That is explained away easily enough: we wanted a term that referred to a property, so we built a term to do it; but we then discovered that terms cannot refer to properties, and thus that 'the property *horse*' cannot do what we built it to do. The really troubling problem is that a number of claims that we want to make about properties appear to require us to refer to them with singular terms. Frege himself was well aware of this awkwardness, but again, in his published work he was fairly dismissive of the problem:

By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realise that in such cases I was relying upon a reader who would be ready to meet me halfway — who does not begrudge a pinch of salt. (Frege 1892a, 192)

There is a sense in which asking your reader to grant you a pinch of salt is completely unobjectionable. It is really nothing more than asking your reader to be charitable, and to accept that in order to keep things brief and readable, you had no choice but to speak a bit loosely. (I would certainly like to ask the readers of this book to add that much salt.) But this only remains reasonable when you *can* speak strictly, when you can state clearly what your views amount to. What the concept *horse* paradox demands, then, is that we find a way of saying everything that we need to say about properties without ever trying to refer to them with singular terms. Frege made some progress on this front in his unpublished (1891–5), and in this book I will try to go further.

Properties as satisfaction conditions

The most important thing we need to say about properties is that predicates refer to them. All we mean by 'property' is something that a predicate can

refer to! But our ordinary way of talking about reference demands that we specify the referent of a given expression with a singular term. We cannot say,

'() is a horse' refers to is a horse,

because that is ungrammatical. To restore grammaticality, we have to say something like this:

'() is a horse' refers to the property *horse*.

But that is exactly what a Fregean realist refuses to say. '() is a horse' is meant to refer to a property, but if 'the property *horse*' refers to anything, then it refers to an object.

To get out of this trouble, Fregean realists need to invent a whole new way of talking about predicate-reference, one which allows us to specify the referent of a predicate *with a predicate*. In Chapter 6, I will argue that we should use,

(S) $\forall y(y \text{ satisfies } x \leftrightarrow Yy)$,

to express predicate-reference. If we do, then this is how we will specify what '() is a horse' refers to:

$\forall y(y \text{ satisfies ' () is a horse' } \leftrightarrow y \text{ is a horse})$.

On this conception, what it is for a predicate to refer to a property is just for it to have a satisfaction condition. And really, properties just are the satisfaction conditions of predicates.

This is clearly a very deflationary conception of properties. It is hard to imagine anyone denying that predicates refer to properties in *this* sense. There is really no room for a substantial debate between nominalists and Fregean realists over the existence of properties (see Chapter 7). And this is not the only debate that Fregean realism would put to bed. Although I think we can say everything that we *need* to say about properties without referring to them with singular terms, there are definitely things that philosophers have *wanted* to say about properties that cannot be re-expressed in this way. But these are all things that we are better off not being able to say. They are the roots of a number of stubborn puzzles in the metaphysics of properties. There will be a reward, then, for finding a way to live with the concept *horse* paradox: it will allow us to dismiss these metaphysical puzzles as mere pseudo-problems, which come up only because we mistakenly speak as if properties were a species of object. Or at least, that is what I will argue in Chapter 10.

I will go even further in Chapters 11–14, and argue that we can extend this conception of properties to cover propositions and states of affairs as well. We can think of states of affairs as zero-place properties, and so as the referents of

zero-place predicates, i.e. sentences.⁴ Now, since we have already signed up to the general idea that properties are satisfaction conditions, that makes states of affairs the satisfaction conditions of sentences, otherwise known as *truth conditions*. A *fact* can then be defined as a truth condition which happens to be satisfied. All of this will be discussed in more detail in Chapter 11.

Things are less straightforward when it comes to propositions, but I will argue that they are also best thought of as zero-place properties (Chapters 12–13). This will lead us to a version of the *identity theory of truth* (Chapter 14): true propositions do not merely correspond to facts; true propositions *are* facts. From this perspective, we will see no need to offer any account of how it is that propositions manage to represent the world, no gap between propositions and the world which needs to be filled by a bit of philosophy.

Williamson on absolute generality

I hope that I have said enough to justify my project on its own terms. However, I would also like to take a moment to talk about how it relates to the wider world of philosophical logic. For a long time, philosophers were deeply suspicious of second-order logic. The chief antagonist was, of course, Quine, who famously declared that second-order logic was set-theory in sheep's clothing: the inference from 'Socrates is wise' to ' $\exists X X(\text{Socrates})$ ' might look like an innocent existential generalisation, but really it introduces a commitment to sets that was not already latent in the original sentence (Quine 1970, 66–8).

These days, however, philosophers look more kindly on second-order logic. The tide was first turned by Boolos (1975, 1984, 1985), but much of the recent interest in second-order logic is due to Williamson's (2003) influential work on absolute generality.⁵ It turns out that if we want to interpret our first-order quantifiers as ranging over absolutely every object, then we cannot think of interpretations themselves as a kind of object, on pain of Russell's Paradox. To see this, consider an interpretation, \mathcal{I} , which assigns the following satisfaction condition to some predicate ' $F()$ ':

$$\forall x(x \text{ satisfies } 'F()' \text{ on } \mathcal{I} \leftrightarrow \neg x \text{ satisfies } 'F()' \text{ on } x).$$

If the quantifier $\forall x\dots x\dots$ ranges over absolutely all objects, and interpretations are objects, then we can infer the following contradiction:

$$\mathcal{I} \text{ satisfies } 'F()' \text{ on } \mathcal{I} \leftrightarrow \neg \mathcal{I} \text{ satisfies } 'F()' \text{ on } \mathcal{I}.$$

⁴ Throughout this book I will mean *declarative sentence* by 'sentence', unless I clearly indicate otherwise.

⁵ Williamson's (2013) work on higher-order modal logic has also been very influential.

Williamson's solution to this problem is simple: he denies that interpretations are objects. Interpretations are not the sorts of things that you quantify over with *first-order* quantifiers. If you want to quantify over interpretations, then you have to use a *second-order* quantifier.

As Williamson is well aware, this solution presupposes that Frege was broadly right about second-order quantification: second-order quantification must not be first-order quantification in disguise; second-order quantification has to be its own thing, a way of quantifying over properties, not objects. This could, I think, be made into a good argument for Fregean realism. But it is not the argument that I will give. My argument will be much more direct: I will focus on the semantic roles of terms and predicates themselves, without making any assumptions about absolute generality. But this convergence of ideas is noteworthy nonetheless. My hope is that those who side with Williamson on absolute generality might look on Chapters 1–9 as providing some of the philosophical underpinnings for their view. Then, in Chapters 10–14, we will see what else Fregean realism can do.