Fregean Predication

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Metaphysics is taking a higher-order turn. More and more metaphysicians are choosing to theorise about properties and propositions in irreducibly higher-order terms.[1] This paper concerns the justification of a higher-order principle called *β-Equivalence*. β-Equivalence is a standard formal assumption, but it is metaphysically controversial. My aim is to provide a novel argument for β-Equivalence, based on a Fregean conception of predication.

1 λ-Abstraction and β-Equivalence

In this section, I will briefly introduce higher-order logic, λ-abstraction and β-Equivalence. For a more detailed account, I highly recommend Bacon 2024.[2]

The higher-order logic that we will be working with is strictly typed. There is one basic type: e. Intuitively, type e terms correspond to natural language names, like 'Socrates' and 'Plato', and should be thought of as referring to objects. If $\tau_1, \ldots, \tau_n$ are types, then $\langle \tau_1, \ldots, \tau_n \rangle$ is the type of predicates which produce formulas from terms of the displayed types, as follows: $\mathbf{a}^{\langle \tau_1,\ldots,\tau_n \rangle}(\mathbf{b}_1^{\tau_1}, \ldots, \mathbf{b}_n^{\tau_n})$.[3] (If some $\mathbf{b}_i$ is not of type τ_i, then $\mathbf{a}^{\langle \tau_1 \ldots \tau_n \rangle}(\mathbf{b}_1, \ldots, \mathbf{b}_n)$ is ill-formed.) Intuitively, $\mathbf{a}^{\langle \tau_1,\ldots,\tau_n \rangle}$ expresses a relation between entities of the displayed types, and $\mathbf{a}^{\langle \tau_1,\ldots,\tau_n \rangle}(\mathbf{b}_1^{\tau_1}, \ldots, \mathbf{b}_n^{\tau_n})$ says that the values of $\mathbf{b}_1, \ldots, \mathbf{b}_n$ (in that order) stand in the relation expressed by $\mathbf{a}$. In the limiting case, when $n = 0$, the rule for building types yields terms of type $\langle\,\rangle$. They are formulas, and should be thought of as expressing propositions.

In the previous paragraph, I moved smoothly from the type of a *term* to the type of an *entity*. An entity of type τ is anything that can be the value of a type τ term. (I will reserve 'object' for entities of type e.) Each entity is assumed to have exactly one type, meaning it can be the value of exactly one type of term. For a defence of this assumption, see Trueman 2021: chs 1–5.

We have a countable infinity of simple constants, and a disjoint countable infinity of variables, of every type. (No term belongs to more than one type.) We also have all of the usual truth-functional operators, which are themselves conceived of as predicates.

[1] For examples, see the papers collected together in Fritz and Jones 2024.

[2] The main differences between Bacon's (2024: ch.3) 'full λ-language' and the higher-order language I describe below are: (i) Bacon's language is based on *functional* types, whereas mine is based on *relational* types; (ii) Bacon permits vacuous λ-abstraction, but I do not (see fn. 7).

[3] Throughout I will use bold fonts as metalinguistic variables ranging over expressions. Strictly speaking, we should use Quine-quotes in connection with these variables — e.g. $\ulcorner \mathbf{a}^{\langle \tau \rangle}(\mathbf{b}^{\tau}) \urcorner$ — but I will omit them for readability. More generally, I will abuse the use/mention distinction whenever doing so is more helpful than confusing.

For example, conjunction is a predicate of type $\langle\langle\,\rangle,\langle\,\rangle\rangle$.[4] There is a distinct identity-predicate for each type τ: $=_\tau^{\langle\tau,\tau\rangle}$. (The superscript indicates the type of the identity-predicate, and the subscript is a helpful reminder that it expresses identity for type τ. Wherever possible, I will omit superscripts and subscripts.) Each identity-predicate is assumed to be reflexive and to obey Leibniz's Law:[5]

Ref: $\mathbf{a}^\tau =_\tau \mathbf{a}^\tau$

LL: $\mathbf{a}^\tau =_\tau \mathbf{b}^\tau \rightarrow (\mathbf{C}^{\langle\rangle} \leftrightarrow \mathbf{D}^{\langle\rangle})$, whenever **C** can be obtained from **D** by replacing some occurrence of **a** with **b**, provided that no variable free in **a** or **b** is bound in **C** or **D**

There are also distinct quantifiers for each type: $\forall_\tau^{\langle\langle\tau\rangle\rangle}$ combines with a type $\langle\tau\rangle$ predicate, and quantifies over type τ entities; roughly, $\forall_\tau \mathbf{F}^{\langle\tau\rangle}$ says that every type τ entity has the property expressed by **F**. These quantifiers behave exactly as you would expect: $\forall_\tau \mathbf{F}^{\langle\tau\rangle}$ entails $\mathbf{F}^{\langle\tau\rangle}\mathbf{a}^\tau$, and the reverse entailment also holds if **a** is arbitrary. Formally, the behaviour of the universal quantifier is governed by these axioms:[6]

UI: $\forall_\tau \mathbf{F}^{\langle\tau\rangle} \rightarrow \mathbf{F}\mathbf{a}^\tau$

Gen: If $\vdash \mathbf{A}^{\langle\rangle} \rightarrow \mathbf{B}^{\langle\rangle}$, then $\vdash \mathbf{A} \rightarrow \forall_\tau \mathbf{x}^\tau \mathbf{B}[\mathbf{x}^\tau/\mathbf{a}^\tau]$, provided that **x** is a variable that does not appear free in **B**, and **a** is a simple constant that appears in **B** but not in **A**

where $\mathbf{B}[\mathbf{x}/\mathbf{a}]$ is the result of substituting a free occurrence of **x** for every occurrence of **a** in **B**. The existential quantifier is governed by the obvious duals of these axioms.

A moment ago, I said that quantifiers are predicates, but in Gen we have a quantifier binding a variable. This wrinkle is usually ironed out by treating $\forall \mathbf{x}^\tau \mathbf{B}$ as an abbreviation for $\forall_\tau \lambda \mathbf{x}^\tau.\mathbf{B}$. The term $\lambda \mathbf{x}^\tau.\mathbf{B}$ is to be read as a complex predicate of type $\langle\tau\rangle$. More generally, the λ-operator is a device for turning formulas into predicates. (Or, at least, this is the standard way of understanding the λ-operator. I will recommend an alternative Fregean understanding in §6.) Let $\mathbf{x}_1^{\tau_1}, \ldots, \mathbf{x}_n^{\tau_n}$ be pairwise distinct variables that all appear free in **A**; $(\lambda \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{A})$ is then a predicate of type $\langle\tau_1, \ldots, \tau_n\rangle$.[7] Intuitively, you might read $\lambda x^e.\mathbf{A}$ as: *is an x such that* **A**. However, it is not clear that there is any acceptable English reading of higher-order λ-terms. Moreover,

[4] Officially, all predications are written in prefix notation, e.g. $\wedge^{\langle\langle\rangle,\langle\rangle\rangle}(p^{\langle\rangle}, q^{\langle\rangle})$; however, when it is more familiar, we may write in infix notation, e.g. $p \wedge q$. As this example also illustrates, I will drop brackets and commas whenever there is no serious risk of ambiguity. Similarly, I will add brackets whenever they make a formula easier to parse.

[5] These schemes should really come surrounded with Quine-quotes (see fn. 3). For example, Ref is not the assertion that every term, $\mathbf{a}^\tau$, is identical to itself (although that is true). Rather, it is a schematic assertion of $\ulcorner \mathbf{a}^\tau =_\tau \mathbf{a}^\tau \urcorner$, for all $\mathbf{a}^\tau$. The same goes for all other schemes presented in this paper. I should also note that the use of a capital **C** and **D** in LL is not significant; it is just a concession to the familiarity of first-order notation, where predicates and sentences are indicated by capitalisation rather than type-superscripts. I will make free use of capitalisation whenever it aids readability.

[6] Strictly speaking, UI and Gen are schemes, but for ease of expression, I will often elide the distinction between an axiom and an axiom-scheme in the main text.

[7] The requirement that $\mathbf{x}_1^{\tau_1}, \ldots, \mathbf{x}_n^{\tau_n}$ be free in **A** prevents us from performing *vacuous* λ-abstraction, e.g. $\lambda x.Fa$. Vacuous λ-abstraction is permitted in many higher-order systems, but it is widely regarded

no matter how we choose to gloss λ-terms informally, their behaviour is ultimately determined by the axioms governing them.

For now, we will focus on just one such axiom:

β-Equivalence: $(\lambda \mathbf{x}_1^{\tau_1} \dots \mathbf{x}_n^{\tau_n}.\mathbf{A})(\mathbf{a}_1^{\tau_1}, \dots, \mathbf{a}_n^{\tau_n}) =_{\langle\rangle} \mathbf{A}[\mathbf{a}_i/\mathbf{x}_i]^{i \le n}$

where $\mathbf{A}[\mathbf{a}_i/\mathbf{x}_i]^{i \le n}$ is the result of simultaneously substituting each $\mathbf{a}_i$ ($i \le n$) for every free occurrence of $\mathbf{x}_i$ in $\mathbf{A}$, provided that no free variable in any $\mathbf{a}_i$ becomes bound in the process.[8] Since β-Equivalence is offered as an axiom of higher-order logic, we may use Gen to universally generalise on any simple constants that appear in its instances.

Here are some examples of β-Equivalence in action:

(1) $(\lambda x.\neg Fx)a = \neg Fa$
(2) $(\lambda x.Fx \lor Gx)a = (Fa \lor Ga)$
(3) $(\lambda X.\neg Xa)F = \neg Fa$
(4) $(\lambda x.Rxb)a = Rab$
(5) $(\lambda xy.Ryx)ba = Rab$

β-Equivalence is a simple and appealing formal assumption. However, when we view it as a metaphysical hypothesis, it can look highly questionable. As if by magic, β-Equivalence conjures new properties and relations out of propositions that seem not to involve them. (1) turns the proposition that a does not have a certain property, *being F*, into the proposition that a does have another property, *being not F*. Similarly, (2) turns the proposition that a has one of two properties, *being F* or *being G*, into the proposition that a has a certain property either way, *being F or G*. (3) is even more peculiar: it turns the proposition that a does not have a certain first-level property, F, into the proposition that F has a certain higher-level property, *not being had by a*. (4), on the other hand, produces a monadic property out of a dyadic relation: it turns the proposition that a bears the relation R to b into the proposition that a by itself has the property *bearing R to b*. And (5) reverses the order of a relation: it turns the proposition that a bears R to b into the proposition that b bears the converse of R to a.

This may already be enough to put off those metaphysicians who have a taste for desert landscapes. But the controversy around β-Equivalence extends beyond that. It is common to think of propositions as complex entities that are somehow built out of properties and objects: the proposition that Fa, for example, is built out of the property F and the object a. This picture motivates the following principle:

Structure: $\forall X^{\langle\tau\rangle} \forall Y^{\langle\tau\rangle} \forall w^{\tau} \forall v^{\tau} (Xw = Yv \leftrightarrow (X = Y \land w = v))$

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as somewhat dubious. (For example, see Dorr 2016: 57-8.) Importantly, the Fregean approach to λ-terms that I lay out in §6 does not extend to the vacuous case.

[8] From now on, this 'no-inadvertent-binding' proviso will be baked into all uses of the $[\dots/\dots]$ notation. In practice, it poses no real obstacle, since we can always work around it just by re-lettering bound variables. This re-lettering presupposes α-Conversion: $\mathbf{A} = \mathbf{B}$, whenever $\mathbf{B}$ can be obtained from $\mathbf{A}$ by replacing some occurrence of $\lambda \mathbf{x}_1^{\tau_1} \dots \mathbf{x}_n^{\tau_n}.\mathbf{C}$ with $\lambda \mathbf{y}_1^{\tau_1} \dots \mathbf{y}_n^{\tau_n}.\mathbf{C}[\mathbf{y}_i/\mathbf{x}_i]^{i \le n}$, provided no $\mathbf{y}_i$ is free in $\mathbf{C}$. I will assume α-Conversion throughout this paper; see fn. 29 for my preferred justification.

But Structure is inconsistent with β-Equivalence.[9]

In light of all this, it becomes pressing to ask what positive justification can be offered for β-Equivalence. The purpose of this paper is to develop a distinctively Fregean justification. (I will also attempt to extend this justification to cover a more general principle, called *β-Conversion*, in §8.) But we should first set the stage by taking a look at the best justification for β-Equivalence already available in the literature.

2 The Argument from Definition

Dorr (2016: 64–6) and Goodman (2016: 174, 2024) have both attempted to justify β-Equivalence by thinking of λ-abstraction as a device for automating explicit definition.[10] We are all familiar with the practice of explicitly defining a predicate in terms of an open formula, as in:

(1) $Fx =_{df} Rxb$

It is crucial to this practice that we always be permitted to substitute an instance of the left-hand side of (1) for the corresponding instance of the right-hand side.[11] So take this example of the reflexivity of propositional identity,

(2) $Rab = Rab$

and substitute Fa for the first occurrence of Rab:

(3) $Fa = Rab$

Now, in (1), we introduced a new bespoke symbol as our defined predicate. But it would be more convenient if we had a notation for automatically introducing defined predicates, without actually needing to stop and give the definition. That, according to Dorr and Goodman, is exactly what λ-abstraction does for us. $\lambda x.Rxb$ should be thought of as a convenient alternative to F that wears its definition on its face. So we may substitute $\lambda x.Rxb$ for F in (3):

[9] For each τ, β-Equivalence is inconsistent with Structure, so long as there are at least two entities of type τ, a^τ and b^τ: by β-Equivalence, $(\lambda x^\tau.x = b)a = (a = b) = (\lambda x^\tau.a = x)b$; but, by hypothesis, $a \neq b$. (And, in case anyone was hoping to exploit any loopholes: there are at least two propositions, since LL implies $A^{\langle\rangle} \neq \neg A$.) Notoriously, Structure is also inconsistent with a weakened version of β-Equivalence, where the propositional identity is swapped for a material biconditional. For discussion, see: Dorr 2016: §6; Goodman 2017: §3.2, 2024: §5.1; Fritz 2023b; Bacon 2023: 231–2, 2024: ch. 11.

[10] Something like this idea is familiar from λ-calculus textbooks (e.g. Hindley and Seldin 2008: 1–2); however, Dorr and Goodman were the ones to develop it as a philosophical justification for β-Equivalence. Goodman (2024) also attempted to justify α-Conversion and η-Equivalence in the same way. (I stated α-Conversion in fn. 8 above, and I will discuss η-Equivalence in §8.) If successful, the Dorr-Goodman argument would *also* justify the standard formation rules for λ-terms, which is notable because Bacon (2023, 2024: pt. III) has explored the possibility of restricting those formation rules.

[11] Here and throughout, I set aside indirect contexts like 'Sharon believes…' and 'Joe hopes…'. It is an interesting question how β-Equivalence should interact with contexts like these; but, then, it is an interesting question how *any* logical axiom should interact with those contexts.

(4) $(\lambda x.Rxb)a = Rab$

This is an instance of β-Equivalence. But there is nothing special about this instance. So, if we think of λ-abstraction as a device for automating explicit definition, then we have a general justification for β-Equivalence.

I think that this *Argument from Definition* is fundamentally along the right lines. Unfortunately, however, there is a serious problem with the argument as it stands. As Dorr (2016: 66) notes, it presupposes that explicit definitions introduce new predicates. But you might instead think of them as introducing mere abbreviations: F is not a predicate in its own right; instead, the whole open formula Fx is shorthand for Rxb.[12] If that is the right way to think about explicit definitions, then λ-abstraction cannot be automated definition, because, by stipulation, λ-terms are predicates.

This is not an idle reservation. There appears to be good reason to think of explicit definitions as mere abbreviations. Explicit definitions are usually assumed to be *non-creative*, in the sense that they do not introduce any new theoretical commitments. But, in a higher-order setting, introducing a new predicate can introduce a new theoretical commitment, because we can existentially generalise on that predicate. For example, we can infer this generalisation from (4):

(5) $\exists X(Xa = Rab)$

But, by itself, (2) does not appear to imply any such higher-order commitment, since it seems to make no mention of any monadic properties.[13]

Importantly, this problem cannot be dodged just by allowing explicit definitions to be creative (i.e. to introduce new commitments). Dorr and Goodman hope to leverage the good standing of explicit definition to justify β-Equivalence. But it is precisely the non-creativity of explicit definition that puts it in such good standing: it never makes sense to object to an explicit definition, since all it ever lets us do is reformulate claims that we could already make in other terms. If we now allowed explicit definitions to be creative, then they would only be as unobjectionable as the new commitments that they introduced.

However, we could save the Argument from Definition if we could find a satisfying way to deny that β-Equivalence really introduces *new* commitments. In the case at hand, we would need to claim that, despite appearances to the contrary, (2) *does* imply (5), all by itself. My aim is to develop a Fregean conception of predication that will allow us to claim exactly that. So my Fregean justification for β-Equivalence can be seen as a way of repairing the Argument from Definition.

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[12] Dorr (2016: 66) conflates the idea that definitions introduce abbreviations with the idea that they introduce *meta-linguistic* abbreviations, which he then dismisses as a mistake. The latter idea is indeed a mistake, but abbreviations do not have to be meta-linguistic.

[13] This objection to the Argument from Definition presupposes the classical quantifier rules, as presented in §1. One way to block this objection would be to adopt weaker 'free' quantifier rules (see Bacon 2024: 100). However, in this paper, I want to find a way of justifying β-Equivalence without weakening the classical rules.

3 Notations and Conceptions

My justification for β-Equivalence is also related to another argument offered by Dorr (2016: 62–3), this time for Involution.

Involution: $\forall p^{\langle\rangle}(p = \neg\neg p)$

Dorr's argument for Involution begins with a notation originally described by Ramsey (1927: 161–2), in which formulas are not negated by prefixing them with a special symbol, $\neg$, but by writing them upside down. So, rather than writing $\neg p$, we would write $\rotatebox[origin=c]{180}{p}$. In this notation, Involution would be reduced to the reflexivity of identity: rather than writing $\neg\neg p$, we would just flip $\rotatebox[origin=c]{180}{p}$ back the right way up, which would give us p again. So, assuming that Ramsey's notation is adequate for metaphysical theorising, we should accept Involution.

I think it is fairly clear that this argument would not move anyone who rejected Involution. They would obviously just deny that Ramsey's notation is adequate, precisely because it prevents us from distinguishing $\neg\neg p$ from p. However, reflecting on Ramsey's notation is still helpful. Ramsey was operating with a Tractarian conception of negation, according to which 'negation reverses the sense of a proposition' (Wittgenstein 1922: 5.2341): p expresses a condition on the world, and is true if that condition is satisfied, false otherwise; $\neg p$ expresses exactly the same condition but swaps around truth and falsity, so that $\neg p$ is false if that condition is satisfied, true otherwise (Wittgenstein 1922: 6.1203). Given this Tractarian conception, Ramsey's notation reflects the relation between a proposition and its negation more perspicuously than the standard notation. So Ramsey's notation is a helpful way of drawing out the consequences of the Tractarian conception. One of these consequences is that Involution is not only true, but *obviously* true. Involution is not the substantive assumption that it appears to be when we write it in the standard notation. It is nothing more than a trivial application of the reflexivity of identity. What we have here, then, is an argument for the triviality of Involution, given the Tractarian conception of negation.

I will attempt to justify β-Equivalence in a exactly the same way. I will lay out a Fregean conception of predication, and describe a notation that perspicuously reflects that conception (or at least reflects it more perspicuously than the standard notation). I will then use this notation to show that β-Equivalence is trivial on the Fregean conception of predication, for just the same reason that Involution is trivial on the Tractarian conception of negation: β-Equivalence is also nothing more than the reflexivity of identity in disguise.

But, before we get going, it would probably be helpful for me to say a little more about what I mean by a 'conception' of predication.[14] In particular, I should be clear that the Fregean conception is not a *formal semantics*. We already know that there are

[14] Thanks to two anonymous reviewers for encouraging me to be clearer on this issue.

formal semantics that validate β-Equivalence, and others that don't.[15] The difficult question is which ones (if any) are right about β-Equivalence. Of course, the *intended* semantics would have to be right, but that is not much help here. On the intended semantics, a refers to a type e object, $\lambda x.Rxb$ expresses a type $\langle e \rangle$ property, and Rab expresses a type $\langle\,\rangle$ proposition. So, the intended semantics will validate $(\lambda x.Rxb)a = Rab$ iff, for every admissible interpretation I, $[\![\lambda x.Rxb]\!]_I^{\langle e \rangle}[\![a]\!]_I^e = [\![Rab]\!]_I^{\langle\,\rangle}$ — where $[\![\ldots]\!]_I^\tau$ maps a type τ term to its value on I. But figuring out whether the intended semantics actually satisfies this condition is precisely where things get controversial.

The last thing we need, then, is *another* formal semantics. What we really need is an *informal* way of thinking about predication which tells us how the intended semantics should treat β-Equivalence. This approach is exactly analogous to the way that many philosophers have used the iterative conception of set to justify various set-theoretic axioms.[16] The iterative conception is not a *formal model* of the sets, but an informal picture of how they are arranged. It is intuitively clear that, if the iterative conception is correct, then every instance of Separation (for example) must be true. Similarly, I will argue that, if the Fregean conception is correct, then every instance of β-Equivalence must be true.

4 The Fregean Conception of Predication

One of Frege's big ideas was that sentences are *multiply-decomposable*.[17] Take the sentence 'Socrates taught Plato', for example. According to Frege, we can parse this sentence into predicate and arguments in multiple ways. He represented the different predicates by deleting their arguments, like this:[18]

(1) __ taught Plato
(2) Socrates taught __
(3) $\_\__1$ taught $\_\__2$
(4) $\_\__2$ taught $\_\__1$

The numerical subscripts on the gaps in (3) indicate that the gaps are *uncoordinated*, in the sense that they may be filled by different names. The subscripts also order the gaps, so that we may distinguish between the predicate's first and second arguments. (4) is just like (3), except the gaps have been ordered the other way round. We can also form a predicate with *coordinated* gaps, which must be filled with the same name. For example, from 'Socrates taught Socrates', we can form:

[15] For the model theory for higher-order logic, see Bacon 2024: pt. IV. Wehmeier (2021) also provides a semantics that validates β-Equivalence, but operates with an understanding of λ closer to the one I recommend below (although see fn. 28 for some residual differences).

[16] For examples, see: Boolos 1971, 1989; Shoenfield 1977; Potter 2004; Paseau 2007; Incurvati 2020: chs 1–2; Button 2021.

[17] See Frege 1879: §9, 1891, 1893: pt. I.

[18] Frege himself used Greek letters (e.g. ξ and ζ) to mark his gaps, but it is helpful to use a slightly more systematic notation.

(5) $\_\_\__1$ taught $\_\_\__1$

The matching numerical subscripts indicate that the gaps in (5) are coordinated, and so (5) is a monadic predicate, despite having two gaps.

Frege thought that each of (1)–(4) is present in 'Socrates taught Plato'. Of course, none is a straightforward *part* of that sentence. They all include gap markers that do not appear anywhere in the complete sentence. (It is even more obvious that (5) is not a part of 'Socrates taught Socrates'.) But (1)–(4) do still appear in 'Socrates taught Plato' in another sense: this sentence could be formed by filling the gaps in each of these predicates with appropriate names. Frege then took the sentence 'Socrates taught Plato' to predicate different things of different people, depending on how we decompose it: it says about Socrates that he taught Plato, but it also says about Plato that Socrates taught him; it says about Socrates and Plato (in this order) that the former taught the latter, but it also says about Plato and Socrates (in *that* order) that the latter taught the former.

Ramsey (1925: 405–6) famously argued that this Fregean conception of predication was incoherent. He assumed that each way of decomposing 'Socrates taught Plato' would correspond to a different proposition, and so took the Fregean picture to imply that 'Socrates taught Plato' expressed many propositions. But, Ramsey thought, that would be absurd: it would lead to a spurious mystery about why all of these propositions necessarily share the same truth-value. However, this is a misunderstanding of the Fregean view. 'Socrates taught Plato' says multiple things *about* Socrates and Plato, both individually and as a pair. But that does not mean that the sentence says *simpliciter* multiple things, in the sense of expressing multiple propositions. The sentence is univocal: it only says that Socrates taught Plato. By *saying that* Socrates taught Plato, it thereby *says about* Socrates that he taught Plato, and *says about* Plato that Socrates taught him.[19]

Here is another way to put the point. The reason that Ramsey thought that the different decompositions of 'Socrates taught Plato' would have to correspond to different propositions is that he accepted something like the Structure principle from §1.[20] However, as we will shortly see, the Fregean picture justifies β-Equivalence, which is inconsistent with Structure. So the Fregean picture requires rejecting the idea that propositions are structured in the way that Ramsey assumed.

This might seem like a surprising thing to say. Frege is often described as identifying propositions with the senses of sentences, and he steadfastly maintained that the senses of the parts of a sentence are parts of the sense of that sentence.[21] However, this

[19] I am here relying on the fact that Socrates and Plato both exist. I will set aside what happens when we say things about (what intuitively appear to be) non-existent objects for another time.

[20] More precisely, Ramsey (1925: 406) thought that p and q must be distinct propositions if 'they have different sets of constituents'. There might initially seem to be some tension between this fine-grained individuation of propositions and Involution, which Ramsey also accepted: *prima facie*, negation is a constituent of $\neg\neg Rab$, but not of Rab. However, Ramsey was a card-carrying Tractarian, and so did not think that negation was ever a constituent of a proposition (Wittgenstein 1922: 4.0312 & 5.4611).

[21] See Frege 1893: §32, 1919: 364–5, 1923: 390.

is really just a confusion of terminology. Frege himself called the senses of sentences *thoughts* (or *Gedanke* in German), and we should not identify our propositions with Frege's thoughts. In this paper, propositions are the values of sentences, and are better thought of as the referents of sentences, not their senses.[22] Frege famously took the referents of sentences to be truth-values, the True and the False, which he conceived of as objects. Most contemporary higher-orderists agree that Frege made two mistakes here: we need to individuate propositions more finely than by their truth-value; and propositions are not objects (type e), but entities of type $\langle\,\rangle$. However, Frege was at least right to select unstructured things as the referents of sentences.[23]

That, then, is an initial sketch of Frege's conception of predication. I am sure that it is very familiar, at least in outline. Pretty much every logic textbook informally introduces predicates in something like Frege's way. However, Frege's predicates-as-gappy-sentences do not usually make an appearance in formal contexts. The closest we get are λ-terms, for example $\lambda x.Txp$.[24] However, given the Fregean conception of predication, the standard λ-term notation is imperspicuous. In a standard λ-calculus, each sentence has a unique decomposition. There is no sense in which the predicate $\lambda x.Txp$ appears in the sentence Tsp. The only predicate to appear in Tsp is T, and so that sentence only expresses a relation between s and p. It does not predicate a property of s in isolation. To do *that*, you need the sentence $(\lambda x.Txp)s$.

If we want a notation that more perspicuously reflects Frege's conception of predication, then we should include his gappy predicates in our formal syntax, instead of relegating them to the antechamber of logic.

5 Higher-Level Predicates

In the previous section, I focussed on *first-level* predicates, i.e. Fregean predicates that have gaps for names. But Frege also made use of *higher-level* predicates, i.e. Fregean predicates that have gaps for other Fregean predicates. The paradigm examples are quantifiers.

We will need to reintroduce some type-indices. Names are still of type e, and gaps for names will be marked as such, e.g. 'Socrates taught $\_\_^e$', which we can formalise as $Ts\_^e$. Since this predicate has just one gap, and it is a gap for a name, it is of type $\langle e\rangle$. We can then form a type $\langle\langle e\rangle\rangle$ Fregean predicate by deleting $Ts\_^e$ from a sentence.

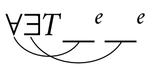
[22] We should also be cautious about identifying 'propositions' in this sense with the 'propositions' that we bear propositional attitudes to. As it happens, I *do* want to make this identification (Trueman 2018, 2021: chs 12–14, 2022), but that is a controversial step that we do not need to take here.

[23] Frege (1892: 159) did experiment with the idea that, in some special sense, the referents of the parts of a sentence are parts of the referent of the sentence, but he eventually rejected it outright (Frege 1919: 365; Frege and Carnap 2004: 87).

[24] The standout exceptions are the Fregean formal systems presented by Wehmeier (2018, 2021). I will diverge from Wehmeier somewhat (see fn. 28), but, for the most part, what follows is intended to build on Wehmeier's work.

For example, from $\exists x^e Tsx$ we can form $\exists x^e \_\_^{\langle e \rangle} x$. This is a *second-level* predicate, i.e. a predicate with a gap for a first-level predicate.

According to Frege, $\exists x^e \_\_^{\langle e \rangle} x$ *is* the first-order existential quantifier. (Frege did not make use of λ-terms in his notation, and so did not think of $\exists x Tsx$ as an abbreviation for $\exists \lambda x.Tsx$.) Importantly, the variables in this Fregean quantifier should not be treated as independent syntactic units. For Frege, the variables in $\exists x^e \_\_^{\langle e \rangle} x$ are there only to indicate that, when we plug a type $\langle e \rangle$ predicate into the quantifier, we thereby close the gaps in that predicate. For example, in $\exists x^e Tsx$, the gap in $Ts\_^e$ is closed by $\exists x^e \_\_^{\langle e \rangle} x$. We need to indicate which gaps are closed by which quantifiers in order to handle multiple-generality: we need to distinguish between $\forall x \exists y Txy$, where $\forall x \_\_^{\langle e \rangle} x$ closes the gap in $\exists y T\_^e y$, and $\forall x \exists y Tyx$, where $\forall x \_\_^{\langle e \rangle} x$ closes the gap in $\exists y Ty\_^e$. Variables are one solution to this problem, but there are others. For example, we could use the Quine-Bourbaki notation, which draws lines between a quantifier and the gaps it closes, as follows:[25]



The Quine-Bourbaki notation is especially clear, but it is also a nightmare to typeset. So we will stick with variables, while bearing in mind that, for a Fregean, they are doing nothing more than the Quine-Bourbaki track-lines.

That is how the Fregean account of second-level predicates is *meant* to go. Unfortunately, however, we have glossed over two noteworthy difficulties. The first is really just a nuisance. On the Fregean model, it is not just the quantifiers that need to indicate which gaps they are closing. *Every* second-level predicate needs to indicate which gaps it is closing. In some cases, this is done just by leaving a name in the gap: if we delete $F\_^e$ from Fa^e, we get $\_\_^{\langle e \rangle} a^e$. However, we will need to use variables whenever there is no name to hand. For example, if we wanted to introduce a simple Fregean predicate of type $\langle \langle e \rangle, \langle e \rangle \rangle$, we would have to write it as something like this: $Rx(\_\__1^{\langle e \rangle} x, \_\__2^{\langle e \rangle} x)$. This is in principle doable — it is how Frege's (1893: §25) concept-script worked — but it certainly makes for a cluttered notation.

The second difficulty is much more serious. In general, first-level predicates are not neatly localised to one spot in a sentence; they are often spread right through the sentence as a whole. But this makes it hard to know what, in general, it means to 'delete' a first-level predicate from a sentence. For example, what exactly do we get when we delete $F\__1^e \wedge G\__1^e$ from $\exists x^e (Fx \wedge Gx)$? It at very least seems a stretch to say that we get $\exists x^e \_\_^{\langle e \rangle} x$.

The difficulties only multiply when consider *third-level* predicates, i.e. predicates that have gaps for second-level predicates. Suppose, for example, we deleted the first-order quantifier $\exists x^e \_\_^{\langle e \rangle} x$ from $\exists x^e Fx$. The result should be a third-level predicate; more precisely, it should be a predicate of type $\langle \langle \langle e \rangle \rangle \rangle$. We might try to write this

[25] See Quine 1981: §12; Bourbaki 1954: ch. 1; Button and Walsh 2018: §1.4; Wehmeier 2018: §4.

predicate as follows: $\_\_^{\langle\langle e\rangle\rangle} x^e Fx$. Note that we have had to keep the variable x^e, to indicate that the gap in $F\_^e$ is closed. However, this variable is not now attached to a second-level predicate, but to a *gap* for a second-level predicate. So we not only have to keep track of which *predicates* are closing gaps in other predicates, but which *gaps* are closing other gaps too. Now, that may just be another nuisance, but things get more serious again. Consider the type $\langle\langle e\rangle\rangle$ predicate we get by deleting $F\_^e$ from Fa^e: $\_\_^{\langle e\rangle} a^e$. We should be able to substitute this predicate into the gap in $\_\_^{\langle\langle e\rangle\rangle} x^e Fx$. But I really have *no* idea what the result of that substitution would look like. (I know what it *should* look like: it should just be Fa. However, I cannot explain how to get that result from general syntactic principles.)

In short, the Fregean procedure of forming predicates by deleting terms from sentences is not clearly defined for all cases. It is straightforward enough when we are just deleting names, but it becomes increasingly difficult to apply as we move up the type hierarchy.[26]

6 λ-Simplification

The serious problems for Fregean predicate-formation are consequences of the fact that Fregean predicates in general are not localised to one spot in a sentence. However, *simple* Fregean predicates are localised, as a special case. A simple type $\langle e\rangle$ predicate still comes with a gap,[27] but in a well designed notation, it always appears in the same place: we can take a simple type $\langle e\rangle$ predicate to be a symbol — call that the *stem* of the predicate — followed by one gap of type e, e.g. $F\_^e$. If we only ever formed type $\langle\langle e\rangle\rangle$ predicates by deleting *simple* type $\langle e\rangle$ predicates, then we could stipulate that to delete a type $\langle e\rangle$ predicate is just to delete its stem. (You should then think of each type $\langle e\rangle$ gap as being immediately followed by a type e gap of its own.)

As an added bonus, if we only ever used simple predicates as arguments to higher-level predicates, then the quantifiers would no longer need to come with variables (or any similar device). Rather than writing $\exists x^e Fx$, we could just write $\exists F$. We here indicate that $\exists$ has closed the gap in $F\_^e$ simply by omitting that gap. This is *not* to

[26] These difficulties are not resolved by the syntax described in *Basic Laws* (Frege 1893: §§21–5). For related criticism of Frege's syntax, see Pickel 2010.

[27] Dummett (1981a: 30–3, 1981b: ch. 16) argued that we must distinguish between simple and complex predicates: complex predicates come with gaps because they are formed by deleting names from whole sentences; but simple predicates are among the basic building blocks of sentences, and so do not come with gaps. However, it seems to me that the properly Fregean view is that sentences, names and predicates are all *coeval*: to understand a simple expression is to understand how it would contribute to the meaning of a sentence in which it appeared; but equally, to understand a sentence is to understand how the simple terms from which it is composed work together to produce its meaning. (For an extended discussion of this point, see Sullivan 2010: §6.) There is a separate question about whether there is a further level of analysis, beneath the name/predicate distinction, which recognises (e.g.) F — without the gap — as a meaningful unit. (According to Sullivan (2010: 116–7) and Potter (2020: ch. 69), this was Ramsey's (1925) key point in 'Universals'.) Fortunately, for present purposes, I can remain agnostic on this issue.

renege on the idea that simple predicates have gaps; it is just that, in this notation, we indicate that ∃ has closed a gap by not marking it out, rather than by writing a variable in it. This notation would be adequate because we would never need to indicate which gaps a quantifier had closed: if we only ever used simple type $\langle e \rangle$ predicates as the arguments to ∃, then there would only ever be one gap that ∃ could have closed. The same would go for all type $\langle\langle e \rangle\rangle$ predicates: all such predicates must close the gaps in their arguments, but we could always indicate that merely by omitting those gaps.

This suggests, then, that an elegant way to accommodate complex Fregean predicates is to introduce a device for *simplifying* them, i.e. for making them behave syntactically as if they were simple. We can repurpose the λ-operator for this task. Take the complex predicate $F\_{}^{e}_{1} \wedge G\_{}^{e}_{1}$. The idea is that we apply the λ-operator to this predicate, as follows: $(\lambda x^{e}.Fx \wedge Gx)\_^{e}$. We then stipulate that $(\lambda x^{e}.Fx \wedge Gx)\_^{e}$ is to express the very same property as $F\_{}^{e}_{1} \wedge G\_{}^{e}_{1}$: all we are doing is gathering up the gaps in the complex predicate; everything else about that predicate is to be left just as it is. We can now treat $(\lambda x^{e}.Fx \wedge Gx)\_^{e}$ as if it were a simple Fregean predicate, with $(\lambda x^{e}.Fx \wedge Gx)$ as its stem. So rather than trying to plug $F\_{}^{e}_{1} \wedge G\_{}^{e}_{1}$ directly into $\exists\_\_^{\langle e \rangle}$, we first simplify it to $(\lambda x^{e}.Fx \wedge Gx)\_^{e}$, and plug that in instead: $\exists \lambda x^{e}.Fx \wedge Gx$.

At this point, our notation looks exactly like a standard λ-calculus. But it is important to be clear that the λ-operator is behaving very differently. We are no longer using the λ-operator to abstract predicates from open formulas. We do *not* apply λx to the open formula $Fx \wedge Gx$. Rather, the λ-operator is now a device for *syntactically simplifying* Fregean predicates: we apply it to the predicate $F\_{}^{e}_{1} \wedge G\_{}^{e}_{1}$, and we get the predicate $(\lambda x^{e}.Fx \wedge Gx)\_^{e}$. We should not think of this process of *λ-simplification* as introducing semantically new predicates into our language; it just syntactically tidies up the complex Fregean predicates that we already had to hand.

With the help of λ-simplification, it is now easy to systematise the hierarchy of Fregean predicates, by presenting a formal syntax. We will use exactly the type-indices that were introduced in §1: e is the only simple type, and whenever $\tau_1, \ldots, \tau_n$ are types, so is $\langle \tau_1, \ldots, \tau_n \rangle$.

We have a countable infinity of simple constants, and a disjoint countable infinity of variables, for each type. No simple constant or variable belongs to more than one type. A simple constant of type $\langle \tau_1, \ldots, \tau_n \rangle$ consists of a *stem*, followed by a sequence of n uncoordinated gaps of the appropriate types, ordered from left to right: e.g. $F^{\langle \tau_1, \ldots, \tau_n \rangle}(\_\_{}^{\tau_1}_{1}, \ldots, \_\_{}^{\tau_n}_{n})$. Variables never include any gaps, and nor do terms of type e or $\langle\,\rangle$. For convenience, though, we will say that every term of type e or $\langle\,\rangle$ is its own stem. Some simple constants are logically privileged, for example $\exists_e^{\langle\langle e \rangle\rangle}\_\_^{e}$ and $\wedge^{\langle\langle\rangle,\langle\rangle\rangle}(\_\_{}^{\langle\rangle}_{1}, \_\_{}^{\langle\rangle}_{2})$; however, they behave syntactically just like any other simple constants.

We then offer the following recursive definition of the well-formed Fregean terms, where $\mathbf{B}^*$ is the stem of $\mathbf{B}$:[28]

[28] This formal Fregean syntax is similar to the one offered by Wehmeier (2021: 526–8), but there are three noteworthy differences: first, Wehmeier did not present the λ-operator as a device for simplifying

(i) Every simple constant of type τ is a simplified term of type τ, and every simplified term of type τ is a term of type τ

(ii) If $\mathbf{A}$ is a term of type $\langle\tau_1,\ldots,\tau_n\rangle$, and $\mathbf{B}_1,\ldots,\mathbf{B}_n$ are simplified terms of type $\tau_1,\ldots,\tau_n$ respectively, then $\mathbf{A}[\mathbf{B}_i^*/\_\__i^{\tau_i}]^{i\leq n}$ is a simplified term of type $\langle\,\rangle$

(iii) If $\mathbf{A}$ is a term of type $\langle\,\rangle$, and $\mathbf{B}_1,\ldots,\mathbf{B}_n$ are pairwise distinct simplified terms of type $\tau_1,\ldots,\tau_n$ respectively, and each $\mathbf{B}_i^*$ occurs in $\mathbf{A}$, then the result of replacing one or more occurrences of each $\mathbf{B}_i^*$ in $\mathbf{A}$ with $\_\__i^{\tau_i}$ is a term of type $\langle\tau_1,\ldots,\tau_n\rangle$, provided no occurrence of any $\mathbf{B}_i^*$ that is being replaced with a gap is embedded within an occurrence of some other $\mathbf{B}_j^*$ that is also being replaced with a gap

(iv) If $\mathbf{A}$ is a term of type $\langle\tau_1,\ldots,\tau_n\rangle$, and $\mathbf{x}_1,\ldots,\mathbf{x}_n$ are pairwise distinct variables of type $\tau_1,\ldots,\tau_n$ respectively, and no $\mathbf{x}_i$ appears in $\mathbf{A}$, then $(\lambda\mathbf{x}_1\ldots\mathbf{x}_n.\mathbf{A}[\mathbf{x}_i/\_\__i^{\tau_i}]^{i\leq n})(\_\__1^{\tau_1},\ldots,\_\__n^{\tau_n})$ is a simplified term of type $\langle\tau_1,\ldots,\tau_n\rangle$; $(\lambda\mathbf{x}_1\ldots\mathbf{x}_n.\mathbf{A}[\mathbf{x}_i/\_\__i^{\tau_i}]^{i\leq n})$ is the stem of that simplified term

(v) Nothing else is a term, or a simplified term, of any type

The sentences — i.e. closed terms of type $\langle\,\rangle$ — of this Fregean syntax are the same as the sentences of the standard syntax. However, the sentences are constructed very differently. Whenever we would have applied λ-abstraction to an open formula in the standard syntax, we now apply *Fregean abstraction* (clause iii) to an appropriate closed sentence, and *λ-simplification* (clause iv) to the resulting Fregean predicate.[29] For example, in the standard syntax, we would have formed $\lambda X^{\langle e\rangle}y^e.Xy\wedge\neg Xa$ simply by attaching $\lambda X^{\langle e\rangle}y^e$ to the open formula $Xy\wedge\neg Xa$. But in this Fregean syntax, we start with an appropriate sentence, such as $Fb\wedge\neg Fa$,[30] apply Fregean abstraction to yield $\_\__1^{\langle e\rangle}\,\_\__2^{e}\wedge\neg\_\__1^{\langle e\rangle}a$, and finally λ-simplify to get $(\lambda X^{\langle e\rangle}y^e.Xy\wedge\neg Xa)(\_\__1^{\langle e\rangle},\_\__2^{e})$.

Fregean predicates; second, Wehmeier's simple predicates and λ-terms do not come with gaps; and third, Wehmeier's syntax never allows us to use a complex predicate *as* a predicate, but only as an argument. For most purposes, these differences would be inconsequential, but they will matter for the Fregean justification of β-Equivalence in §7.

However, having said that, Fregeans might still find it useful to consider a restricted syntax that does not allow complex predicates to appear *as* predicates. This restricted syntax yields all the same terms as the unrestricted syntax, but more complex terms are always derived from simpler ones. (In Dummett's (1981b: 271–2) terminology, the restricted syntax exclusively *analyses* terms into their *constituents*, whereas the unrestricted syntax also *decomposes* terms into their *components*.) This feature of the restricted syntax proves helpful when developing a Fregean definition of compositionality (see Trueman forthcoming).

[29] It should be noted that the Fregean conception of predication immediately justifies α-Conversion (see fn. 8): when λ-simplifying a complex predicate, clause (iv) requires us to introduce fresh variables for our λ-operator to bind, but *which* fresh variables is entirely arbitrary; whichever we choose, the resulting λ-simplification will be stipulated to express the same property as the original complex predicate. Here is another way to make the same point: it would have been even more perspicuous on the Fregean conception to use Quine-Bourbaki track-lines to connect λ-operators to gaps, and in that notation, distinct formulas are never α-variants.

[30] This sentence itself is constructed by repeated appeals to (ii): first we form Fa and Fb by applying the simple predicate $F\_^{e}$ to the simple names a^e and b^e; then we form $\neg Fa$ by applying the simple predicate $\neg^{\langle\langle\,\rangle\rangle}$ to Fa; finally, we form $\wedge(Fb,\neg Fa)$ — i.e. $Fb\wedge\neg Fa$ in infix notation — by applying the simple predicate $\wedge^{\langle\langle\,\rangle,\langle\,\rangle\rangle}(\_\__1^{\langle\,\rangle},\_\__2^{\langle\,\rangle})$ to Fb and $\neg Fa$.

Given the Fregean conception of predication outlined in §4, this Fregean notation is more perspicuous than the standard one. Each sentence is multiply-decomposable. For example, Tsp is a result of filling T_p with s, but it is *also* a result of filling $Ts\_$ with p, which reflects the fact that Tsp says something about the referent of s, *and* says something about the referent of p. Moreover, we stipulate that every λ-simplification is to express the very same property as the predicate it simplifies. So $\lambda x.Txp$ expresses precisely the property that Tsp predicates of the referent of s.

7 A Fregean Justification for β-Equivalence

We can now offer an argument for β-Equivalence, based on the Fregean notation that I have just laid out. Start with this instance of the reflexivity of propositional identity:

(1) $Rab = Rab$

In the Fregean notation, Rab can be formed in multiple ways. One of the ways is to complete the predicate $R\_^{e}b$ by writing a in the gap. $R\_^{e}b$ is a complex predicate, but we can λ-simplify it, if we wish: $(\lambda x.Rxb)\_^{e}$. Crucially, this process of λ-simplification is stipulated merely to move the gap in $R\_^{e}b$; $(\lambda x.Rxb)\_^{e}$ is to express exactly the same property as $R\_^{e}b$.[31] So there cannot be any semantic difference between writing a into the gap in $(\lambda x.Rxb)\_^{e}$, and writing it into the gap in $R\_^{e}b$. We are therefore permitted to substitute $(\lambda x.Rxb)a$ for the first occurrence of Rab in (1):

(2) $(\lambda x.Rxb)a = Rab$

This is an instance of β-Equivalence. But there is nothing special about this instance. So, in general, β-Equivalence amounts to nothing more than the reflexivity of identity.

Back in §2, I presented a problem for the Dorr-Goodman Argument from Definition: β-Equivalence appears to be creative in a sense that explicit definitions must not be. In particular, $(\lambda x.Rxb)a$ seems to introduce a new commitment to a property that was not already implicit in Rab. However, β-Equivalence only appears to be creative when the λ-operator is read as a device for turning open formulas into predicates. From a Fregean perspective, Rab *already* predicates a property of the object a: it can be decomposed into the predicate $R\_^{e}b$ and the name a, and so it says about the object a that it bears the R-relation to the object b. From this point of view, then, (1) already implies:

(3) $\exists X(Xa = Rab)$

[31] This crucial stipulation is essentially *meta-linguistic*. (Thanks to an anonymous reviewer for urging me to be explicit on this point.) If we wanted to express it in the object-language, we would have to write something like this:

* $\lambda x.Rxb =_{\langle e\rangle} R\_^{e}b$

But that is ill-formed in the syntax I have laid out. You need to λ-simplify $R\_^{e}b$ before you can input it as an argument to any higher-level predicate.

Syntactically speaking, it is convenient to facilitate the derivation of (3) by using λ-simplification to turn (1) into (2). But that is not because (2) supplies us for the first time with a property to witness (3). That property was there all along in (1). It is just that, by λ-simplifying $R\_^{e}b$, we make that predicate syntactically easier to manipulate as an argument to higher-level predicates.

At this point, though, you might worry that the Fregean conception of predication has not really eliminated the creativity that was involved in β-Equivalence, but merely relocated it. In particular, you might worry that it now appears in the very process of forming a Fregean predicate by deleting simplified terms from a sentence. We start by plugging the names a and b into the simple predicate $R\_{}_1\_{}_2$, and we get the sentence Rab. This sentence expresses a relation between the objects a and b. But then we form the complex predicate R_b, and now Rab predicates a new monadic property of a alone.[32]

However, this worry is misplaced. It presupposes that, before we formed R_b, Rab *only* expressed a relation between a and b. But, according to the Fregean conception, Rab *always* predicated a monadic property of a as well. By saying about a and b that the former bears R to the latter, the sentence Rab thereby says about a that it bears R to b. Of course, you could understand this sentence without *realising* that it predicates a property of a. To figure that out, you would need to observe that you can form R_b, and that might not have crossed your mind. Indeed, there is a sense in which $R\_{}_1\_{}_2$ is privileged over R_b: if you come to understand Rab in the ordinary compositional way, then you *must* recognise that it can be formed by filling $R\_{}_1\_{}_2$ with a and b. (Frege (1879: §7) used exactly this fact to explain how quantified logic manages to be informative.[33]) But this privilege extends only as far as our *recognition* of what Rab predicates of which objects. Whether we realise it or not, the sentence Rab predicates a monadic property of a right from the start. That is what makes the notation described in §6 so perspicuous: it draws no syntactic distinction between applying $R\_{}_1\_{}_2$ to a and b together, and applying R_b to a by itself.

Alternatively, you might worry that the creativity of β-Equivalence now appears in the stipulation that every λ-simplification is to express the same property as the predicate it simplifies.[34] Applied to the case at hand, this is the stipulation that $(\lambda x.Rxb)\_$ is to express the same property as R_b. But, crucially, $(\lambda x.Rxb)\_$ and R_b are both terms of exactly the same type: they are both predicates of type $\langle e \rangle$. And, in general, there can be nothing objectionably *creative* about stipulating that one term is to have the same semantic value as another term of the same type. This is what distinguishes the Fregean justification from simply stipulating β-Equivalence at the outset, without invoking the Fregean conception. The latter stipulation introduces semantically new *predicates* by reference to semantically familiar *formulas*; for example, stipulating (2) introduces a property that maps a to the proposition that Rab. Introducing predicates

[32] Thanks to Nick Jones for pressing this objection.

[33] Also see Dummett (1981b: 271–2) on the distinction between *analysing* a sentence into its *constituents*, and *decomposing* it into mere *components*.

[34] Thanks to an anonymous reviewer for questioning me on this point.

like *that* may be creative; but merely stipulating that one predicate is to express the same property as another plainly is not.

It is very tempting to use Frege's (1884: §64) famous metaphor here, and say that the left-hand-side of β-Equivalence 're-carves the content' of the right-hand-side. But it is important to be clear about what the point of this metaphor would be.[35] In a sense, *anyone* who accepts β-Equivalence thinks that the two sides share a content: after all, β-Equivalence is an *identity*. What matters is that, given the Fregean conception of predication, β-Equivalence is not just true, but *trivially* true. There is no semantic difference between the two sides of (2), and what little syntactic difference there is hardly amounts to much: $R\_^{e}b$ has a gap in the middle, and $(\lambda x.Rxb)\_^{e}$ has a gap at the end. From a Fregean point of view, (2) is just the reflexivity of identity in disguise. So the left-hand-side of (2) does not 're-carve' the right-hand-side by virtue of having a significantly different syntactic or semantic structure. The left-hand-side merely makes notationally salient one of the decompositions that the right-hand-side already made available.[36]

Importantly, though, I have not yet said anything about whether you *should* accept the Fregean conception of predication that underwrites this argument for β-Equivalence. I will return to this question in §9, where I will briefly discuss one broad meta-metaphysical outlook that would make the Fregean conception especially attractive. Before that, however, I want to consider a potential shortcoming of the Fregean justification for β-Equivalence.

8 β-Conversion and Functionality

You might reasonably worry that the Fregean argument for β-Equivalence proves too little. Consider the following identification:

(1) $(\lambda x^{e}.(\lambda y^{e}.Fy \wedge Gy)x \wedge Hx) =_{\langle e \rangle} (\lambda x^{e}.Fx \wedge Gx \wedge Hx)$

Intuitively, β-style reasoning should imply (1). However, (1) expresses an identity between *properties*, whereas β-Equivalence yields identities between *propositions*. So, if we want to prove (1), then we will need to generalise β-Equivalence. The standard way of doing that is to adopt β-*Conversion* (where each $\mathbf{t}_i^{\tau_i}$ may be open or closed):

[35] It is also important to be clear that the kind of content re-carving involved in β-Equivalence provides no direct support for the kind of content re-carving that Frege (1884: §64) was actually talking about, namely the kind allegedly involved in abstraction principles. (See Dummett 1981b: 332–3, 1991: 173–6; Hale 1997: §3. For criticism of neo-Fregean abstractionism, see Trueman 2014.)

[36] As a Fregean, it is also tempting to say that the two sides of β-Equivalence have the same sense. However, whether that is the right thing to say will depend on exactly how you conceive of sense. If you think of sense as the dimension of meaning that is relevant to logic and nothing else, then I think you should say that the two sides share a sense. However, if you tie sense more closely to what is involved in our *understanding* a sentence, then I think you should not say it: as I have stressed above, you can understand the right-hand-side without spotting all the ways in which it can be decomposed. (Potter (2020: 94) argues that, despite not emphasising it in his published writings, Frege had the former view of sense. Dummett (1981a) famously championed the latter view.)

β-Conversion: $\mathbf{A} = \mathbf{B}$, whenever $\mathbf{B}$ can be obtained from $\mathbf{A}$ by replacing some occurrence of $(\lambda \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1}, \ldots, \mathbf{t}_n^{\tau_n})$ with $\mathbf{C}[\mathbf{t}_i/\mathbf{x}_i]^{i \leq n}$

We can derive $\lambda x.Fx \wedge Gx \wedge Hx$ from $\lambda x.(\lambda y.Fy \wedge Gy)x \wedge Hx$ by replacing $(\lambda y.Fy \wedge Gy)x$ with $Fx \wedge Gx$, and so β-Conversion implies (1).

However, this appeal to β-Conversion is problematic from a Fregean point of view. Why should we be permitted to substitute $Fx \wedge Gx$ for $(\lambda y.Fy \wedge Gy)x$? The Fregean answer is supposed to be that $(\lambda y.Fy \wedge Gy)\_$ is just a λ-simplification of $F\_{1} \wedge G\_{1}$, and so plugging x into one of these predicates is equivalent to plugging it into the other. But the Fregean syntax presented in §6 does not permit the construction of open terms. Variables are only ever introduced in the process of λ-simplifying a predicate, and so every variable is bound by a λ-operator. (This is not a quirk of the syntax; it follows directly from the key Fregean idea that the λ-operator applies to predicates, not formulas.) As a result, neither $(\lambda y.Fy \wedge Gy)x$ nor $Fx \wedge Gx$ is well-formed in the Fregean syntax.[37]

The Fregean argument for β-Equivalence does not, then, immediately generalise into an argument for β-Conversion. In principle, I suppose a Fregean might just accept this limit on what they can justify, but that would be fairly disappointing. Fortunately, however, there is a way for Fregeans to bridge the gap. Although β-Equivalence does not imply (1), it does imply:

(2) $\forall z((\lambda x.(\lambda y.Fy \wedge Gy)x \wedge Hx)z =_{\langle\rangle} (\lambda x.Fx \wedge Gx \wedge Hx)z)$

A Fregean could then license the move from (2) to (1) by identifying *co-functional* relations, i.e. relations which map the same arguments to the same propositions:

Functionality: $\forall X^{\langle \tau_1,\ldots,\tau_n\rangle} \forall Y^{\langle \tau_1,\ldots,\tau_n\rangle}$
$$(\forall z_1^{\tau_1} \ldots \forall z_n^{\tau_n}(X(z_1,\ldots,z_n) =_{\langle\rangle} Y(z_1,\ldots,z_n)) \rightarrow X =_{\langle \tau_1,\ldots,\tau_n\rangle} Y)$$

This strategy can be generalised: for any closed $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} = \mathbf{B}$ is implied by β-Conversion only if it is implied by β-Equivalence plus Functionality.[38]

[37] Of course, Fregeans can still talk about open terms, whenever they find it convenient (for example, when giving a succinct statement of β-Equivalence): an 'open term' is any string of symbols that *would* be a closed term if closed terms were substituted for all of the free variables. It is just that, for a Fregean, an open term is no more a kind of term than a seahorse is a kind of horse. So Fregeans can understand β-Conversion in exactly the way it is intended. Their problem is finding a way to justify it, so understood.

[38] Here is a sketch of a proof by induction. If $\mathbf{B}$ can be obtained from $\mathbf{A}$ in the way described by β-Conversion, I will call $\mathbf{B}$ a *β-reduct* of $\mathbf{A}$. I will also assume that we may universally generalise on any simple constant in an instance of β-Equivalence; this is permitted because β-Equivalence is an axiom-scheme, and so a simple constant still counts as 'arbitrary' even if it appears in an instance of that scheme.

The base case: Let $\mathbf{A}$ and $\mathbf{B}$ be closed terms, such that $\mathbf{B}$ is a β-reduct of $\mathbf{A}$. Also assume that the relevant $(\lambda \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1}, \ldots, \mathbf{t}_n^{\tau_n})$ does not appear embedded within any λ-terms in $\mathbf{A}$. In that case, β-Equivalence implies $\mathbf{A} = \mathbf{B}$.

The inductive case: Assume $\mathbf{X} = \mathbf{Y}$ whenever $\mathbf{Y}$ is a closed β-reduct of $\mathbf{X}$, and the relevant $(\lambda \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1}, \ldots, \mathbf{t}_n^{\tau_n})$ occurs embedded within k λ-terms in $\mathbf{X}$. Now let $\mathbf{A}$ and $\mathbf{B}$ be closed terms, such that $\mathbf{B}$ is a β-reduct of $\mathbf{A}$, but the relevant $(\lambda \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1}, \ldots, \mathbf{t}_n^{\tau_n})$ occurs

So, if Fregeans accepted the axiom of Functionality, they could provide an argument for β-Conversion. However, Functionality is a controversial axiom. Part of what makes Functionality so controversial is that it identifies relations whenever they happen to map the same *actual* arguments to the same propositions. It is at least conceivable, though, that some relations differ only by mapping *possible* arguments to different propositions. But we can easily accommodate relations like that by weakening Functionality as follows:[39]

Modalised Functionality: $\forall X^{\langle\tau_1,\dots,\tau_n\rangle}\forall Y^{\langle\tau_1,\dots,\tau_n\rangle}$
$$(\Box\forall z_1^{\tau_1}\dots\forall z_n^{\tau_n}(X(z_1,\dots,z_n) =_{\langle\rangle} Y(z_1,\dots,z_n)) \rightarrow X =_{\langle\tau_1,\dots,\tau_n\rangle} Y)$$

There is an interesting question about which notion of necessity we should take $\Box$ to express in this principle. However, I will not pursue that question here. For present purposes, I will just assume that $\Box$ expresses the *broadest* (objective) necessity, in the sense that $\Box\mathbf{A}$ implies that $\mathbf{A}$ is necessary in every (objective) sense.[40]

Crucially, if we adopt Modalised Functionality as a logical axiom, then we can still derive β-Conversion from β-Equivalence.[41] We are always permitted to necessitate the consequences of logical axioms. So, when the axioms of β-Equivalence and Modalised Functionality imply that two properties are co-functional, we can immediately infer that they are *necessarily* co-functional.

This derivation of β-Conversion essentially relies on treating β-Equivalence and Modalised Functionality as *logical axioms*, not mere truths. But that is something that Fregeans are well placed to justify. This should hopefully already be clear in the case of β-Equivalence. The Fregean argument for β-Equivalence does not rely on any assumptions about how the world happens to be, or even how it must be, but only on assumptions about the language we are using to describe the word in any case.

The Fregean conception of predication also provides a principled justification for accepting Modalised Functionality as a logical axiom. (That is no surprise, since Frege (1891) himself conceived of properties and relations, or *concepts* in his terminology,

embedded within k+1 λ-terms in $\mathbf{A}$. (For ease, we will assume that $(\lambda\mathbf{x}_1^{\tau_1}\dots\mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1},\dots,\mathbf{t}_n^{\tau_n})$ appears only once in $\mathbf{A}$. This could be achieved simply by re-lettering bound variables.) Let $\lambda\mathbf{y}_1^{\sigma_1}\dots\mathbf{y}_m^{\sigma_m}.\mathbf{A}^*$ be the largest λ-term in $\mathbf{A}$ that contains $(\lambda\mathbf{x}_1^{\tau_1}\dots\mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1},\dots,\mathbf{t}_n^{\tau_n})$. Now consider $\mathbf{A}^*[\mathbf{c}_i/\mathbf{y}_i]^{i\leq m}$, for arbitrary simple constants $\mathbf{c}_1^{\sigma_1},\dots,\mathbf{c}_m^{\sigma_m}$. $\mathbf{A}^*[\mathbf{c}_i/\mathbf{y}_i]^{i\leq m}$ is a closed term, and $(\lambda\mathbf{x}_1^{\tau_1}\dots\mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1},\dots,\mathbf{t}_n^{\tau_n})[\mathbf{c}_i/\mathbf{y}_i]^{i\leq m}$ appears embedded within only k λ-terms in $\mathbf{A}^*[\mathbf{c}_i/\mathbf{y}_i]^{i\leq m}$. So $\mathbf{A}^*[\mathbf{c}_i/\mathbf{y}_i]^{i\leq m} = \mathbf{A}^*[\mathbf{C}[\mathbf{t}_j/\mathbf{x}_j]^{j\leq n}/(\lambda\mathbf{x}_1^{\tau_1}\dots\mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1},\dots,\mathbf{t}_n^{\tau_n})][\mathbf{c}_i/\mathbf{y}_i]^{i\leq m}$ is an instance of the inductive assumption. β-Equivalence therefore implies that the relation expressed by $\lambda\mathbf{y}_1^{\sigma_1}\dots\mathbf{y}_m^{\sigma_m}.\mathbf{A}^*$ is co-functional with the relation expressed by $\lambda\mathbf{y}_1^{\sigma_1}\dots\mathbf{y}_m^{\sigma_m}.\mathbf{A}^*[\mathbf{C}[\mathbf{t}_j/\mathbf{x}_j]^{j\leq n}/(\lambda\mathbf{x}_1^{\tau_1}\dots\mathbf{x}_n^{\tau_n}.\mathbf{C})(\mathbf{t}_1^{\tau_1},\dots,\mathbf{t}_n^{\tau_n})]$. It then follows by Functionality that these terms express the very same relation. Substituting the latter term for the former in $\mathbf{A}$ yields $\mathbf{B}$, and so we may infer $\mathbf{A} = \mathbf{B}$.

[39] Modalised Functionality is still somewhat controversial (for related discussion, see Fritz 2023a: §3); however, it is justified on the Fregean conception of predication, as I argue below.

[40] For defences of the assumption that there is a broadest necessity, see: Williamson 2016; Bacon 2018, 2024: ch. 7; Bacon and Zeng 2022; Fritz 2023b. However, it should be noted that Fritz and Bacon (but not Bacon and Zeng) presuppose β-Conversion.

[41] This can be demonstrated by a minor variation on the proof given in fn. 38.

as functions. I think that Frege took his functional conception too far,[42] but I take Modalised Functionality to be a kernel of truth.) We should think of the predicate '_ taught Plato' as *saying of* an object that it taught Plato. More generally, we should think of first-level predicates as *saying things of* objects. This way of describing what first-level predicates do is more-or-less inevitable: '__ taught Plato' is one name short of a sentence, and so it must be one reference to an object short of expressing a proposition.[43] Moreover, first-level predicates always come with gaps, even when we do not explicitly mark them out. So, even in a sentence identifying two properties, like $F =_{\langle e \rangle} G$, we should think of $F\_^{e}$ and $G\_^{e}$ as saying things of objects. At this point, the most natural way to understand $F =_{\langle e \rangle} G$ is as identifying what $F\_^{e}$ says of x with what $G\_^{e}$ says of x, for all possible x. (Indeed, it is hard to imagine what *else* $F =_{\langle e \rangle} G$ might mean for a Fregean.) This immediately leads to Modalised Functionality for type $\langle e \rangle$ properties, and the same kind of reasoning motivates Modalised Functionality for the other types.[44]

Of course, all of this would *still* be disappointing, if you had hoped to justify β-Conversion without committing to Modalised Functionality. It is, then, worth observing that there is an independent argument from β-Conversion — taken as its own basic axiom — to Modalised Functionality. This argument relies on two subsidiary axioms of higher-order logic. The first is entirely standard:

η-Equivalence: $\lambda \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{A}^{\langle \tau_1,\ldots,\tau_n \rangle}(\mathbf{x}_1, \ldots, \mathbf{x}_n) =_{\langle \tau_1,\ldots,\tau_n \rangle} \mathbf{A}$, provided that no $\mathbf{x}_i$ is free in $\mathbf{A}$

The purpose of η-Equivalence is to eliminate redundant λ-terms. Here are some examples:

(3) $\lambda x^{e}.F^{\langle e \rangle}x =_{\langle e \rangle} F$
(4) $\lambda x^{\langle e \rangle}.\exists_e x =_{\langle \langle e \rangle \rangle} \exists_e$
(5) $\lambda x^{e} y^{e}.(\lambda w^{e} v^{e}.Fw \wedge Fv)(x, y) =_{\langle e,e \rangle} \lambda w^{e} v^{e}.Fw \wedge Fv$

[42] Frege took properties to be functions to truth-values, and he conceived of truth-values as objects (type e). As I mentioned in §4, I think that this incorporates two mistakes: we need to individuate the values of sentences more finely than by truth-value, and they belong to type $\langle\,\rangle$, not type e.

[43] Compare Dummett 1981b: 167–8.

[44] Kripke (2005: 1025 fn. 45) famously used the idea that λ-terms express functions to justify β-Equivalence. Now that I have adopted Modalised Functionality, you might wonder just how different my justification ends up being from Kripke's. There are two points to make here:

(a) All that I am taking from the functional model is a criterion of individuation for relations. By itself, Modalised Functionality does not imply β-Equivalence. (Nor does the stronger Functionality principle.) Kripke was, then, relying on much more of the functional model than I am, and more than I think can be justified by the Fregean conception of predication.

(b) Although I appealed to Modalised Functionality in my argument for β-Conversion, I did not appeal to it in my argument for β-Equivalence. So, if you like the Fregean conception of predication but think I am wrong to see it as providing a justification for Modalised Functionality, then you still at least have a justification for β-Equivalence.

It is easy to give a Fregean justification for η-Equivalence: in every instance of the scheme, the left side is a (redundant) λ-simplification of the right side. But, even setting this Fregean justification aside, η-Equivalence is widely regarded as uncontroversial. It is certainly very hard to imagine anyone accepting β-Conversion but rejecting η-Equivalence.[45]

The second subsidiary axiom is Involution:

Involution: $\forall p^{\langle\rangle}(p = \neg\neg p)$

Involution is a much more controversial axiom than η-Equivalence, but it strikes me as extremely plausible. Moreover, the Ramsey-Dorr argument from §2 shows that Involution is trivial, given a Tractarian conception of negation. Finally, to my knowledge, the best argument against Involution is that it is inconsistent with Structure;[46] however, as I mentioned in §1, Structure is also inconsistent with β-Conversion, and we have assumed that already.

The reasoning in this argument will be abductive, not deductive. β-Conversion, η-Equivalence and Involution do *not* jointly entail Modalised Functionality. However, what I hope to show is that these axioms cohere better with Modalised Functionality than without it.

Imagine a world that violates Modalised Functionality. In particular, imagine that $F^{\langle e\rangle}$ and $G^{\langle e\rangle}$ are distinct but necessarily co-functional properties: $\Box\forall y(Fy = Gy)$. By Involution and β-Conversion, it also follows that $\Box\forall y(Fy = Gy = (\lambda x.\neg\neg Fx)y)$. But, importantly, these principles do not settle the identity of $\lambda x.\neg\neg Fx$: as far as they are concerned, $\lambda x.\neg\neg Fx$ could be identical to F, to G, or to neither.[47] And η-Equivalence is also no guide here, since it does not apply to terms of the form $\lambda x.\neg\neg Fx$.[48] So while our premises require that we choose a property necessarily co-functional with F as the value of $\lambda x.\neg\neg Fx$, they do not dictate which of them we should choose.

To fix ideas, suppose that F is chosen as the value of $\lambda x.\neg\neg Fx$. Intuitively, though, it should still be possible for there to be *another* operator, λ', which also satisfies (its analogues of) β-Conversion and η-Equivalence, but which chooses G as the value of $\lambda' x.\neg\neg Fx$. However, as Bacon (2024: 68) demonstrates, β-Conversion and η-Equivalence[49] together 'pin down' the behaviour of the λ-operator, in the following sense: if λ and λ' both satisfy β-Conversion and η-Equivalence, then

[45] See Salmon 2010: 453–4; Dorr 2016: 65; Bacon 2024: 65–6.

[46] Or, at least, Involution is inconsistent with Structure if, unlike Ramsey (fn. 20), we treat $\neg$ and $\Box$ as terms: $(\neg\neg\Box P) = (\Box P)$, but $\neg \neq \Box$, and $(\neg\Box P) \neq (\Box P)$.

[47] The assumption that $\Box$ expresses the *broadest* necessity is essential at this point.

[48] η-Equivalence does imply $\lambda y.(\lambda x.\neg\neg Fx)y = \lambda x.\neg\neg Fx$, and so if it were somehow already fixed that $\lambda y.(\lambda x.\neg\neg Fx)y = F$ (for example), it would follow that $\lambda x.\neg\neg Fx = F$. But, of course, the point is that β-Conversion, η-Equivalence and Involution also fail to uniquely single out the value of $\lambda y.(\lambda x.\neg\neg Fx)y$, or of any other λ-term that η-reduces to $\lambda x.\neg\neg Fx$.

[49] Bacon (2024: 66) actually appeals to a principle that generalises on η-Equivalence as β-Conversion generalises on β-Equivalence. However, Bacon's proof only relies on η-Equivalence.

$\lambda \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{A} =_{\langle \tau_1,\ldots,\tau_n \rangle} \lambda' \mathbf{x}_1^{\tau_1} \ldots \mathbf{x}_n^{\tau_n}.\mathbf{A}$. So, in particular, $\lambda' x.\neg\neg Fx$ must be identical to $\lambda x.\neg\neg Fx$.

This is a very peculiar situation. λ would satisfy β-Conversion and η-equivalence no matter whether F or G were chosen as the value chosen for $\lambda x.\neg\neg Fx$. But if λ and λ' both satisfy those axioms, then they are not permitted to choose their values differently: $\lambda' x.\neg\neg Fx$ must have the same value as $\lambda x.\neg\neg Fx$. The best way to avoid this peculiarity — without restricting the axioms of β-Conversion, η-Equivalence or Involution — is to adopt Modalised Functionality.[50] That way, there is never any choice over the value of a λ-term: β-Conversion can only ever permit *one* property to be the value of $\lambda x.\mathbf{A}$.

I take this to be a good abductive argument for Modalised Functionality. So anyone who (like me) is happy to accept η-Equivalence and Involution as higher-order axioms should not bristle at the fact that the Fregean justification for β-Conversion relies on Modalised Functionality.

9 Fregean Deflationism

Over the course of this paper, I have tried to offer a justification for β-Equivalence, and even β-Conversion, on the basis of a Fregean conception of predication. As I noted at the end of §7, that Fregean conception is not *compulsory*. You could choose to reject it, and thereby undermine my justification for β-Equivalence. Whether to accept or reject the Fregean conception must be decided on the same wide-reaching, broadly abductive grounds that we rely on to decide whether to adopt any way of thinking. I will not try to argue here that, in the final assessment, you should adopt the Fregean conception, although I do think the fact that it provides a justification for β-Equivalence speaks in its favour. Instead, I want to end by briefly considering one kind of meta-metaphysical orientation that makes the Fregean conception especially attractive.

Looked at in a certain way, higher-order logic can appear radically inflationary. According to Quine (1948: 21), you can answer the question *What exists?* in just one word: *Everything*. But in a higher-order setting, that is just scratching the surface. There is a whole type-hierarchy of properties and relations, and each one has its own bespoke quantifiers. Philosophers who restrict themselves to first-order logic, on the other hand, look like models of moderation: some of them believe in things that they call 'properties' and 'relations', but those are all still objects.[51]

[50] You might have thought you could also avoid this particular problem just by stipulating that, for all F, $F = \lambda x.\neg\neg Fx$. However, that would miss the point. If Modalised Functionality fails then, even if λ happens to conform this stipulation, it should still be possible for there to be *another* operator, λ', which does not, without violating β-Conversion, η-Equivalence or Involution.

[51] Sider (manuscript) understands higher-order logic in this inflationary way, and expresses serious doubts about it so understood.

However, despite these initial appearances, a number of proponents of higher-order logic seem to take it in a *deflationary* spirit. In some sense, understanding claims about properties and propositions in higher-order terms is meant to make them less metaphysically loaded. For example, it is sometimes claimed that nominalist scruples about the existence of properties do not get any grip in a higher-order setting.[52]

Now, it is very hard to explain exactly what this deflationary approach to higher-order logic comes to. But we can make a start by thinking of properties as the *satisfaction conditions* of predicates:[53] when we use a monadic first-level predicate, for example, we say something of an object; and first-level properties are just the things that can be said of objects. On this kind of view, there is no gap between meaningfully talking about objects, and predicating properties of them.

I think that anyone who adopts this kind of deflationary view of properties should find the Fregean conception of predication attractive.[54] It would seem very strange for a deflationist to think that there is an important difference between saying that Socrates killed Socrates, and saying about Socrates that he killed himself. Or, to put it another way, it would be strange for them to think that committing yourself to a property of *suicide* is to take an extra step, which in principle you might refuse to take, over and above committing yourself to a relation of *killing*.

I would, then, recommend the Fregean conception of predication at least to those philosophers who do not think of higher-order logic as a grand speculative metaphysic of previously unimagined entities, but as a tool for deflating philosophical talk about properties and propositions.[55]

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52 See: Rayo and Yablo 2001; Trueman 2021: ch. 7.

53 See Trueman 2021: ch. 6.

54 Many philosophers have associated Frege with some sort of meta-metaphysical deflationism. However, this is typically in connection with the idea that he once floated (Frege 1884: §§62–5) of defining function-terms with abstraction principles. (See Hale and Wright 2001, 2009; Schiffer 2003: ch. 2; Thomasson 2015: §3.2.) As noted in fn. 35 above, nothing in the Fregean conception of predication provides any direct support for this kind of abstractionism.

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