Eliminating Identity: a reply to Wehmeier

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1. Introduction

In the *Tractatus* (5.53–5.5353) Wittgenstein claimed that there is no identity relation. Recently Wehmeier (2012) has tried to show that Wittgenstein was right. He argues that identity is a problematic (putative) relation, and that all mention of that relation can be dispensed with. In this note I will show that if identity really is problematic then despite Wehmeier's efforts, we do not yet have the means to eliminate it.

2. Wehmeier on how and why to live without identity

Why is Wehmeier worried about identity? Relations are the sorts of things that can, well, relate things. But identity cannot relate *things*: identity only holds between an object and itself. Following Wittgenstein (1922: 5.53–5.533), Wehmeier proposes to eliminate the identity sign in favour of a convention governing the quantifiers: from the range of a bound variable 'x', all objects are excluded that are values of variables occurring free within the scope of the quantifier that binds 'x'; similarly, 'x' cannot take as value the referent of any individual constant within the scope of the quantifier that binds it. Wehmeier calls a logic with such a restriction and without an identity sign, 'Wittgensteinian predicate logic', which I will abbreviate as 'WL'.

Wehmeier has proved that WL is *expressively equivalent* to first-order logic with identity (FOL$^=$): for any sentence in FOL$^=$ there is a sentence in WL which has exactly the same models under the semantics of WL as the original has under the standard semantics, and *vice versa*. His proof comes in three steps. First he repeats Hintikka's (1956) result that WL is expressively equivalent to FOL$^=$ if we assume that there are no individual constants. He (2008: 372–3) then proves that the same is true when we include individual constants, so long as no distinct individual constants co-refer. Finally, he (2008: 374–6) shows that if we add a new binary predicate to WL, the triple bar '$\equiv$', then WL is expressively equivalent to FOL$^=$ even if we allow distinct constants to co-refer. The intended reading of '$a \equiv b$' is 'The constants "a" and "b" co-refer'; the FOL$^=$ sentence '$a = b$' is to be translated into WL as '$a \equiv b$'.

Wehmeier takes these results to show that identity is dispensable. But as we will see, there is more to showing that identity is dispensable than showing that WL and FOL$^=$ are expressively equivalent.

3. Necessity and the triple bar

As Wehmeier (2012: 774) recognises, there is a problem with translating '$a = b$' as '$a \equiv b$'. Take the sentence,

(1) Hesperus is Phosphorus.

(1) looks to be necessarily true. But Wehmeier would have us formalise (1) as,

(2) $h \equiv p$.

(2) is only contingently true: Mars and Jupiter could have been called 'Hesperus' and 'Phosphorus'.

Wehmeier (2012: 774–5) offers a solution to this problem. He suggests that underlying our intuition that (1) is necessarily true is the intuition that one object cannot be two objects. He then claims that with a 'modicum of second-order logic' we can express the claim that Hesperus and Phosphorus are one object as follows:

(3) $\exists F(Fh \wedge Fp \wedge \neg\exists y\exists z(Fy \wedge Fz))$.

(3) says that there is some property which h and p both have but which does not have more than one instance. Wehmeier takes (3) to say that Hesperus and Phosphorus are one object because he thinks it is a logical truth that every object has a *singleton property*, i.e. a property which it and it alone has. Wehmeier argues that we think (1) is necessarily true because we tend to read it as (3), which Wehmeier claims is necessarily true.

However, Wehmeier's appeal to second-order logic undermines his project. If it is a logical truth that every object has a singleton property then it is a logical truth that no two objects share all of their properties. In that case, we can define identity in an ordinary second-order logic in the familiar way:

(4) $\forall x\forall y(I(x,y) \leftrightarrow \forall F(Fx \leftrightarrow Fy))$.

But how can identity be a problematic relation if we have the means to *explicitly define* it?

Wehmeier (2012: 774 fn. 21) does consider this point, and gives the following response. When we translate (4) into a second-order WL, we get the sentence

(5) $\forall x(I(x,x) \leftrightarrow \forall F(Fx \leftrightarrow Fx)) \wedge \forall x\forall y(I(x,y) \leftrightarrow \forall F(Fx \leftrightarrow Fy))$

'$\forall F(Fx \leftrightarrow Fx)$' is a tautologous condition, 'Taut(x)', and given that it is a logical truth that every object has a singleton property, '$\forall F(Fx \leftrightarrow Fy)$' is a contradictory condition in WL, 'Cont(x, y)'. Wehmeier argues as follows:

> $I(x, x)$ is just an awkward and superfluous alternative notation for Taut(x), and $I(x, y)$ for Cont(x, y). The stipulation [i.e. (5)] that was intended to introduce *one* binary relation, identity, has in fact picked out *two* relations: the universal unary relation, represented by Taut(x), and the empty relation Cont(x, y). It's notational legerdemain, as it were, to use the same predicate letter in the two conjuncts of [(5).] (Wehmeier 2012: 773)

But this response is dialectically irrelevant. The fact that (4) becomes (5) in WL will impress us only if we have been convinced to use WL. Wehmeier tries to convince us to use WL by arguing that identity is a problematic relation. But the objection under consideration is that identity cannot be problematic if we have the means to define it. Wehmeier has not, then, done anything to disarm this objection. Of course, that is not yet to say that this objection has been vindicated; to do that, we must look more closely at Wehmeier's worry about identity.

4. Why should identity be problematic?

Wehmeier's mistrust of identity stems from its inability to relate *objects*: all it can do is relate an object to itself. Wehmeier (2012: 768) makes this worry sharper by introducing what he calls the Wittgensteinian Arity Principle (WAP): the arity of a relation R is the maximal number of objects that can possibly be related by R. According to WAP, identity is not a binary relation: the maximal number of objects that identity can relate is one. Identity is really a unary relation, i.e. a property; in particular, it is the property of being self-identical.

If we accept WAP, we cannot admit a binary identity relation. But why should we accept WAP? Well, according to Wehmeier (2012: 768), 'WAP yields the correct result for all standard examples of relations, with the sole exception of the contentious case of identity'. However, WAP does not *always* give the right answer. Consider any contradictory property you like, say *is mortal and is not mortal*. No object can have this property. There is, then, no maximal number of things it can relate, and so WAP denies that it has an arity.[1] But that is wrong: *is mortal and is not mortal* is a unary relation!

[1] To be clear, WAP entails that *is mortal and is not mortal* has no arity, not that it has an arity of zero. According to WAP, a 0-ary relation is a relation that can relate zero objects. Let's say that when a relation relates some objects, that relation and those objects together form a fact. In this terminology, a 0-ary relation is a relation which can form a fact without any additional objects; that is, a 0-ary relation is itself a fact. The relation *is mortal and is not mortal*, on the other hand, can never be used to form a fact, with or without the help of any ancillary objects.

WAP must be modified in some way to deal with contradictory properties. And in fact, Wehmeier's (2012: 772–3) own remarks suggest such a modification. As it stands, WAP has no problem dealing with the complements of contradictory properties, i.e. tautological properties; WAP tells us that *is mortal or is not mortal* is a unary relation, just as it should. Wehmeier might, then, amend WAP as follows: a pair of complementary relations must share an arity, in particular they must share the highest arity that WAP assigns to either relation in the pair. Once WAP is so amended, *is mortal and is not mortal* will count as a unary relation after all.[2]

This amendment raises an obvious problem for Wehmeier. Take the relation of diversity, i.e. the relation that a bears to b just in case a and b are distinct. According to WAP, diversity is a binary relation. But isn't diversity the complement of identity? If so, the amended version of WAP will declare identity to be binary also. However, Wehmeier (2012: 772–3) preempts such an objection. He points out that a relation does not by itself determine its complement: it does so only relative to a background domain. So far, this is quite right. The complement of R, R^c, is defined as follows:

(6) $\forall x \forall y (R^c xy \leftrightarrow \neg Rxy)$

The relation defined by (6) will depend on the ranges of the variables 'x' and 'y'. Let R be diversity. In ordinary first- or second-order logic, where no restrictions are placed on the values that 'y' can take, (6) will produce identity as R^c. But as Wehmeier observes, (6) will not produce identity in WL; rather, it will produce the empty relation. However, taken as a response to our current objection — that when we amend WAP in the way suggested, identity comes out as a binary relation — this observation is dialectically irrelevant in just the way that Wehmeier's earlier response was. The fact that (6) defines the empty relation rather than identity in WL will impress us only if we have been convinced to use WL. Wehmeier wants to convince us to use WL by using WAP to show that there can be no binary identity relation. However, WAP has now been amended, and we will see a tension between this amended version of WAP and the existence of a binary identity relation only if we are *already* using WL. As it stands, then, nothing has been said to show that we cannot happily accept the amended version of WAP, continue to use an ordinary predicate calculus, and recognise the existence of a binary identity relation.

Alternatively, Wehmeier may wish to restrict WAP so that it does not have to deal with contradictory properties at all. Of course, it would be unacceptably *ad hoc* to simply restrict WAP to non-contradictory properties. However, there is a nearby principled restriction: Wehmeier could restrict WAP to *atomic* relations. (There are very good reasons to worry about the whole notion of atomicity, but I will set them aside here.) The arities of non-atomic relations, including contradictory ones, would then be determined by the arities of the atomic relations from which they are constructed and the manner of

[2]Thanks to an anonymous referee for suggesting this line of response.

their construction.

If WAP is restricted to atomic relations then, obviously enough, it will threaten the binary status of identity only if identity is atomic; otherwise it will have nothing to say about the arity of identity. But now recall that the suggestion made at the end of the last section was to define identity with (4). If we do, identity will surely be non-atomic. So this restricted version of WAP also poses no obstacle to the existence of a binary identity relation as defined by (4).

Still, WAP is not the only reason to worry about identity. Indeed, although Wehmeier (2012: 761–2) takes his WAP-based worry to be Wittgensteinian, it plainly was not the *only* thing troubling Wittgenstein about identity. According to the *Tractatus*, all necessary truths are tautologies. Identity was problematic for Wittgenstein because true identity claims would have to be necessarily true, but he did not think that they could be analysed as tautologies. In fact, Wittgenstein (1922: 5.5302) presented this worry as a direct response to the idea that we could define identity with (4). His objection was that while it may be false to say that some pair of objects share all of their properties, it is not contradictory to do so.

However, Wehmeier cannot appeal to this Wittgensteinian objection to defining identity with (4). To repeat, Wehmeier claimed that it is a logical truth that every object has a singleton property. It follows immediately from this claim that it is a logical truth that no two objects share all their properties, and hence that (4) is an adequate definition of identity. So agreeing with Wehmeier's claim that it is a logical truth that every object has a singleton property *already* requires dismissing Wittgenstein's worry.

5. Co-referring constants

To summarise: Wehmeier's ambition is to convince us both that a binary identity relation is philosophically problematic, and that we can dispense with it. In dispensing with identity, he claimed that it is a logical truth that every object has a singleton property. However, if this claim is right then it turns out that a binary identity relation is not problematic after all.

At this point Wehmeier may wish to retrace his steps. As we saw in §2, he presented his elimination of identity in three stages. First he assumed that there were no individual constants, then he introduced constants on the condition that no distinct constants co-refer, and finally he waived that condition. It was only at that final stage that Wehmeier ended up appealing to singleton properties: he appealed to them in order to deal with the fact that (1) seems to be necessarily true but (2) is only contingently true. He may, then, try to avoid such an appeal by convincing us either that we do not need to allow distinct constants to co-refer, or even that we do not need to use constants at all. In this section I will consider the former option, and in the next I will discuss the latter.

Let us suppose that Wehmeier allows us to use individual constants but installs the rule that no distinct individual constants may co-refer. In that case, he will not need to appeal to singleton properties in his translation of '$a = b$': it can be translated into WL as 'Cont(a, b)' (Wehmeier 2008: 373). However, introducing this rule generates a new problem. Suppose that we are trying to live by the rule; also suppose that, like certain ancient astronomers, we do not know whether Hesperus is Phosphorus. In this case we will not know whether we should introduce one name for Hesperus and another for Phosphorus, or just use one name for both of them. That is, we will not know how to follow the rule at hand.

We can make this point more vivid by comparing the above situation with the one we find ourselves in when we allow distinct constants to co-refer. Now we can introduce one name for Hesperus, 'h', and another for Phosphorus, 'p', even if we do not know whether Hesperus is Phosphorus. Our ignorance will manifest itself in an ignorance about the truth-values of certain sentences: '$h = p$' in FOL$^=$ and '$h \equiv p$' in WL. But when distinct constants are not allowed to co-refer, our ignorance does not concern the truth-values of certain sentences. Assume that we still have access to the triple bar even though no two constants may co-refer. In that case, we can write '$h \equiv p$', but our problem is not that we do not know whether this sentence is true. We know that *if* we have followed the rules of the language properly then this sentence cannot be true: to be true two different constants would have to co-refer. Our problem is precisely that we do not know how to follow the rules: we do not know whether Hesperus is Phosphorus and so we do not know how many names to introduce.

So far I have been speaking under the false supposition that we do not know whether Hesperus is Phosphorus. But there is undoubtedly some object a and some object b such that it is an open question whether a is b. Moreover, a language in which distinct constants cannot co-refer would be problematic even if there were no such a and b. To use a familiar metaphor, one object can be presented in different ways: we can see a star at one position in the evening and at another in the morning. It is at least an *intelligible* thought that we have mistaken one object under two different presentations for two different objects. We should, then, be able to express this thought, even if only to assert its negation. And when we allow distinct constants to co-refer, we can: in FOL$^=$ we can say '$a = b$', and in WL we can say '$a \equiv b$'. But when we refuse to allow distinct constants to co-refer, we cannot give voice to this thought.

Wehmeier may reply by digging in his heels and insisting that no one object can be presented in multiple ways. If he does he will be endorsing a Tractarian doctrine (1922: 4.243). But it is important to recognise that this aspect of the *Tractatus* is bound up with Wittgenstein's deeply problematic project of analysis. It would be obviously absurd to deny that complex objects, like stars and mountains, are presented in many different ways. Wittgenstein avoids this absurdity by insisting that so-called 'complex objects' are not *really* objects, and are to be analysed away in favour of *simple* objects. And it does not seem quite as absurd to say that no one simple object can be presented in more than one way, if only because we know so little about what a simple object is.

Very few philosophers today endorse a Tractarian programme of analysis. Moreover, even if there is some idealised sense in which we could analyse away complex objects in favour of their simple counterparts, we plainly have no idea how to actually complete such an analysis. Until we do, we are stuck at the level of complex objects and so will be forced to use a language in which distinct constants may co-refer. So there is a clear sense in which an elimination of identity that involves insisting that no distinct constants may co-refer is not an elimination for the likes of *us*.

6. Variables with values

What if Wehmeier tries the more radical response of suggesting that we do not need to use individual constants at all? In the last section we saw that if one object can be presented in multiple ways, we cannot live by the rule that no two constants may co-refer. This observation should induce in us a further worry about Wehmeier's convention regarding quantifiers and variables. According to it, no bound variable 'x' may take as value the value of any variable that appears free in the scope of the quantifier that binds it. When we assign values to variables, they come to behave essentially like individual constants. So a rule telling us that (in certain contexts) distinct variables cannot be assigned the same value is just a version of the rule telling us that we cannot introduce distinct yet co-referring individual constants. We can no more live by such a restriction on our use of variables than we can on our use of constants.

In more detail: Suppose we spot a star in the evening, a, and a star in the morning, b, and we are not sure whether a is b. Now consider the WL sentence '$\exists x \exists y(Fx \wedge Fy)$'. In this case we do not know whether we can assign b to 'y' after we have already assigned a to 'x'. That is, we do not know how to use the second quantifier in '$\exists x \exists y(Fx \wedge Fy)$'.

Again, it is helpful to contrast the above situation with the one we find ourselves in when we use $\mathrm{FOL}^{=}$. Our ignorance over whether a is b does not stop us from knowing how to *use* the sentence '$\exists x \exists y(x \neq y \wedge Fx \wedge Fy)$': we can assign '$x$' and '$y$' any values we like, including a and b, without worrying about whether we have ended up assigning them the same one. Rather, our ignorance concerns the truth-value of the open sentence '$x \neq y$' when we assign a to 'x' and b to 'y'. Now return to the WL sentence '$\exists x \exists y(Fx \wedge Fy)$'. Here our ignorance is not (merely) about the truth-value of a sentence. Let's suppose that we know that a and b satisfy 'Fx'. In that case, we know what truth-value '$Fx \wedge Fy$' would receive if we were to assign a to 'x' and b to 'y': it would be true. Our problem is that we do not know whether this assignment is permissible: we do not know whether this assignment is one that gets a say in determining the truth-value of '$\exists x \exists y(Fx \wedge Fy)$'.

But if we do not know what assignments to measure the truth of '$\exists x \exists y(Fx \wedge Fy)$' by, then we do not know how to properly use that sentence.

What we can do is talk about WL in a metalanguage which allows distinct variables to take the same value. (That is exactly what I am doing!) In such a metalanguage we can say that '$\exists x \exists y(Fx \wedge Fy)$' is true in WL iff there is some a and some b such that a is not b and '$Fx \wedge Fy$' is true when we assign a to 'x' and b to 'y'. But that is not to say that we know how to use the quantifiers in WL. What we know how to do is *use* the quantifiers of our metalanguage to *talk about* the quantifiers in WL.

7. Conclusion

The general conclusion of this paper can be put as follows. Either it is a logical truth that every object has a singleton property, or it is not. If it is, then the identity sign is undoubtedly eliminable, but this is not an exciting result: we will be able to explicitly define identity, and we can always eliminate explicitly defined terms. Moreover, when we define identity in this way, it becomes unproblematic. If, on the other hand, it is *not* a logical truth that every object has a singleton property, then the identity relation may well be problematic. However, we will be unable to use Wehmeier's method of eliminating that relation: to do so we would need to occupy an idealised perspective to which no one object can be presented in a number of different ways.[3]

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