Quantification 1
The problem of multiple generality

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What is a quantifier?

- All, some, most, there is one, there are two, there are three...
- 'Everything is F' — '$\forall x Fx$' — is true just in case every object in the domain is F. 'Something is F' — '$\exists x Fx$' — is true just in case at least one object in the domain is F

Course outline

1. The problem of multiple generality
2. Frege's treatment of the quantifiers
3. Formal semantics for the quantifiers
4. Substitutional quantification and second-order quantification

Today's lecture

What is multiple generality?

- Any sentence featuring one quantifier within the scope of another is an example of multiple generality
- For example, 'Everyone admires someone'
- In your 1A formal logic course you were taught to translate this sentence into QL as '$\forall x \exists y$ admires(x, y)'
- This way of treating sentences featuring multiple generality has its origins in Frege's *Bergriffsschrift*. The only difference between how we formalise 'Everyone admires someone' today and how Frege did it then is notational. (We are very lucky that Frege's notation did not become standard!)
- We are so comfortable with multiple generality today that it can be difficult to grasp the enormity of Frege's intellectual achievement. The point of this lecture is to give some sense of how momentous Frege's treatment of quantification was

Before Frege

- Before Frege there was Aristotle, writing in the 4th Century BC
- Aristotle was the father of the syllogistic, a simple form of quantification theory. (We will see just how simple later on)
- An example of a syllogism:

All As are Bs
All Bs are Cs
So, All As are Cs

Categorical form

- Syllogisms deal with sentences written in "categorical form"
- S is P
  Subject / Copula / Predicate
  Socrates is mortal
- So central to the Aristotelian tradition was the claim that all sentences should be rewritten to reveal their categorical form
- Quantifiers take the form of noun phrases (all/some + general term) and are treated as subjects
- Treating quantifiers as nouns is a mistake. Who does 'someone' refer to? When we write 'Socrates is a philosopher and Socrates is wise', we say two things of one man: Socrates. 'Someone is a philosopher and someone is wise', on the other hand, can be true even if no one person is both a philosopher and wise. But we will set these concerns aside

Monadic predicates

- Nowadays we would say that Aristotle's syllogistic is a type of *monadic logic*
- In your 1A formal logic course, you were introduced to predicates. Predicates are expressions of the form '$F(x_1, x_2 ... x_n)$' which we can turn into sentences by replacing all of the variables 'x_1', 'x_2'... x_n with terms. (More on this conception of predicates next week!)
- A *monadic* (or one-place) predicate is a predicate with only one variable, like 'x is a horse'
- A *dyadic* (or two-place) predicate is a predicate with two variables, like 'x is the father of y'. A *triadic* (or three-place) predicate is a predicate with three variables, and so on. In general we call predicates with more than one variable *polyadic*

Monadic logic

- To say that Aristotle's syllogistic is a monadic logic is to say that it only features monadic predicates
- Recall that all sentences are to be rewritten in "categorical form", i.e. in the form 'S is P'
- Sentences in categorical form apply a predicate to exactly one subject (although that subject may itself be general)

Relations are ideal

- Relations are treated as "ideal", i.e. as in principle eliminable
  - Athena admires Odysseus
  - Athena is an admirer of Odysseus
- The predicate 'x is an admirer of Odysseus' is treated *en bloc* without any internal complexity
- 'x is an admirer of Odysseus' is not treated as the result of filling one of the blanks in the relational expression 'x is an admirer of y' with the name 'Odysseus'. It is just treated as an unstructured predicate 'x is an admirer-of-Odysseus'

The problem of multiple generality

- We are finally in a position to appreciate the problem of multiple generality
- Aristotle cannot account for the clearly valid argument

Some goddess admires some citizen of Ithaca
All citizens of Ithaca are mortals
So, some goddess admires some mortal

The problem of multiple generality cont.

- He would have us first rewrite the sentences of that argument into their categorical form:

Some goddess is an admirer of some citizen of Ithaca
All citizens of Ithaca are mortals
So, some goddess is an admirer of some mortal

- Aristotle would then treat this argument as of the form

Some A is B
All Cs are D
So, some A is E

- It hardly needs to be said that this is invalid

Diagnosis

- The problem is that categorical form is blind to the significant structural differences between predicates like 'x is mortal' and ones like 'x admires Odysseus'
- The reason for this is that the analysis does not bring out the relational form of sentences like 'Athena admires Odysseus'. Consequently it cannot deal with quantifies nested within predicates
- In other words, it cannot deal with multiple generality

Is logic trivial?

- There is a long tradition of thought according to which logic is somehow *trivial*
- In fact, the word 'trivial' comes from the word 'trivium'. The trivium was the syllabus first taught to students in medieval universities. It was comprised of grammar, logic and rhetoric
- Kant famously insisted that logic was "not ampliative"; roughly, a logical inference never takes us further than where we started. This is one of the reasons that Kant thought that arithmetic was not analytic
- But what exactly do we mean by saying that logic is, or is not, trivial?

Decidability

- By "a logic" I will mean a formal language with some inference rules defined over it. PL and QL are logics in this sense
- One of the things that logicians do is compare the "strengths" of various logics
- One way of measuring the strength of a logic is to ask whether it is *decidable*
- Roughly, a logic **L** is decidable iff we could in principle program a computer to tell us of any given sentence of **L** in a finite period of time whether or not that sentence is a logical truth according to **L**
  (Note: this is not the same as being able to give us *all* of the logical truths of **L** in a finite period of time)

Decidability cont.

- One of the things we might mean when we say that a logic is trivial is that it is decidable
- This makes good sense: if logic is decidable then we could just grind out the consequences of any claim in an unthinking manner; in that sense, the logic took us nowhere new
- But not all logics are decidable
- Propositional logic (PL) is decidable
- First-order logic (QL and its extension $QL^=$) is not decidable
- Monadic logic (which is just the same as QL except we can only use monadic predicates) is decidable

Triviality and objects

- Can we give some philosophical story about why monadic logic is trivial?
- Monadic logic can be thought of as the logic of relations of containment between concepts. This is how we think of it when we use Venn diagrams
- But when we move to polyadic logic, featuring sentences with multiple generality, our notion of logic involves *objects*
- Plausibly, it is this introduction of objects which marks the move from the trivial to the non-trivial
- See Potter *Reason's Nearest Kin* §1.6

The triviality of Aristotle's logic

- Recall that Aristotle's logic was a monadic logic
- Consequently, Aristotle's logic is trivial when we measure triviality in decidability
- For a long time Aristotle was taken to have, in essence, finished logic
- Perhaps this gives us some understanding as to why people took logic to have been trivial
- Next week we will turn to Frege's invention of the non-trivial polyadic logic