The Philosophy of Physics

Lecture Seven

General Relativity

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General Relativity

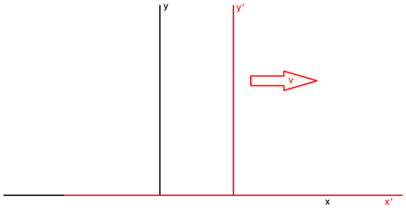
Special Relativity — A Very Quick Reminder

- Special Relativity is built on two postulates:
  1. *The Relativity Postulate*: the laws of nature are the same in all inertial frames of reference
  2. *The Light Postulate*: the speed of light (in a vacuum) is a constant: c

Special Relativity — A Very Quick Reminder

- These two postulates are inconsistent if we stick to the **Galilean transformations** between inertial frames of reference

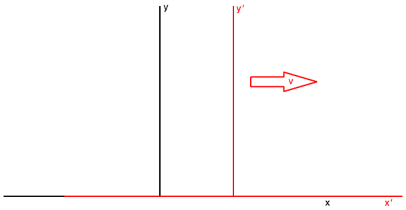
- $t' = t$
- $y' = y$
- $z' = z$
- $x' = x - vt$



Special Relativity — A Very Quick Reminder

- We have to replace the Galilean transformations with the following **Lorentz transformations**:

- $t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$
- $y' = y$
- $z' = z$
- $x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$



What's so Special about Special Relativity?

- These Lorentz transformations tell us how to move from one **inertial** frame of reference to another
  - If an object is at point (x, y, z, t) according to inertial frame F, what point will it be on according to inertial frame F', which is moving relative to F on the x axis at v?
  - The Lorentz transformations tell you how to figure this out
- But the Lorentz transformations can only take us from one inertial frame to another
- SR tells us nothing about what happens when we consider accelerating frames
- SR is special in the sense that it only deals with the special class of inertial frames

SR and Gravity

- SR also has nothing to say about gravity
- In fact, it is **very** difficult to reconcile the traditional, Newtonian conception of gravity with SR
- According to Newton, the strength of the gravitational attraction between two bodies is inversely proportional to the square of the distance between them
  - But in SR, different frames of reference will measure different distances between two bodies!
- According to Newton, gravity was a force acting instantaneously at a distance
  - But in SR, simultaneity is relative, and so different frames will disagree over whether two events occurred at the same instant (i.e. simultaneously)

The General Theory of Relativity

- GR was developed to fill in these gaps
  - It would allow us to work with accelerating frames of reference
  - It would provide us with a relativistic theory of gravity
- Does this mean that SR is **wrong**?
- No!
  - It's just that the applications of SR are limited to special cases: inertial frames where the effects of gravity are negligible

The General Theory of Relativity

No fairer destiny could be allotted to any physical theory, than that it should of itself point out the way to the introduction of a more comprehensive theory, in which it lives on as a limiting case.

(Einstein, Relativity, p. 77)

General Relativity

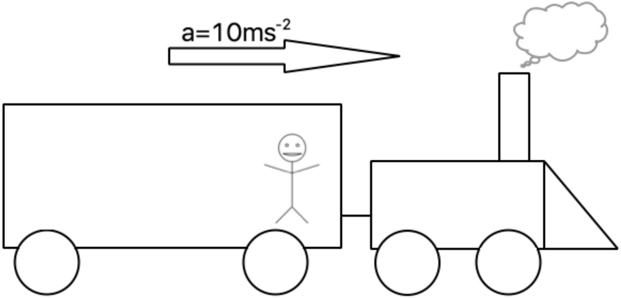
The General Relativity Principle

- In SR, we have *The Postulate of Relativity*:
  - The laws of nature are the same in all **inertial** frames of reference
- This is a special relativity, restricted only to inertial frames
- In GR, we want to have a *General Relativity Principle*:
  - The laws of nature are the same in **all** frames of reference

An Obstacle to General Relativity

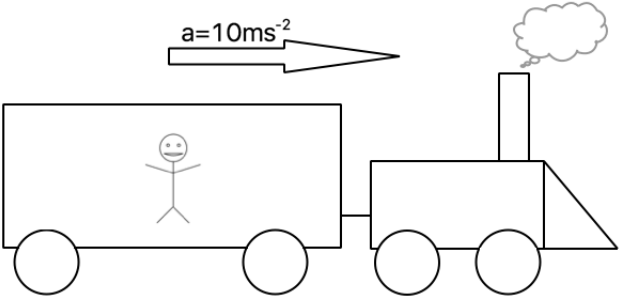
- Since Lecture 2, we have been familiar with the idea of **inertial effects**
- When an object accelerates, it experiences measurable physical forces
- As a result, we want to insist that non-inertial frames of reference **are** physically distinguished from inertial ones

An Obstacle to General Relativity



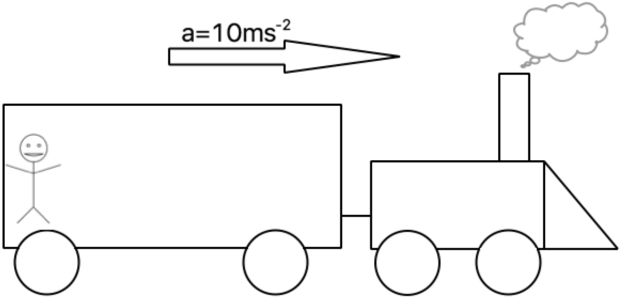
- An observer is standing in a carriage of an accelerating train

An Obstacle to General Relativity



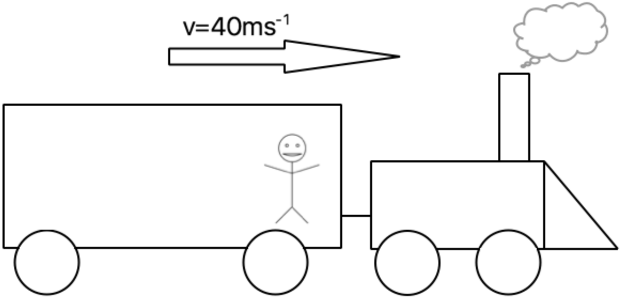
- As the train accelerates, the observer is flung towards the back of the carriage

An Obstacle to General Relativity



- Eventually, the observer hits the back of the carriage, which stops them going any further

An Obstacle to General Relativity



- If the train had been moving with a constant velocity, then the person would not have been moved

An Obstacle to General Relativity

- It seems, then, that we can perform an experiment to figure out if we are in an inertial frame or an accelerating one
- But if the laws of physics were exactly the same in all frames (inertial and accelerating), this shouldn't be possible!
- The first big challenge for GR is to overcome this problem

General Relativity

Two Kinds of Mass

- In Newton's laws of motion, there are **two** kinds of mass:
  - Inertial mass
  - Gravitational mass

Inertial Mass

- The mass of a body as measured by its resistance to acceleration
- Newton's Second Law $F = ma$
  - F is the resultant force acting on the body
  - m is inertial mass
  - a is acceleration

Gravitational Mass

- The mass of a body as measured by its gravitational effects on other bodies
- Newton's Law of Universal Gravitation: $F = G\frac{M_A M_B}{d^2}$
  - F is the gravitational force between A and B
  - G is the gravitational constant
  - M_A is the gravitational mass of A, and M_B is the gravitational mass of B
  - d is the distance between A and B

Gravitational Mass

- We can simplify things by focussing on a single object A in some gravitational field
- Then we can say that the gravitational force acting on A has a magnitude propositional to A's gravitational mass
- $F = M_A g$
  - g measures the strength of the gravitational field at A's location
  - Near the surface of Earth, $g \approx 9.8ms^{-2}$

Relating Inertial and Gravitational Mass

- $F = ma$ (m is inertial mass)
- $F = Mg$ (M is gravitational mass)

- Hence $ma = Mg$
- Hence $\frac{M}{m} = \frac{a}{g}$

Relating Inertial and Gravitational Mass

If now, as we find from experience, the acceleration is to be independent of the nature and the condition of a given body and always the same for a given gravitational field, then the ratio of the gravitational to the inertial mass must likewise be the same for all bodies. By a suitable choice of unit we can thus make the ratio equal to unity. We then have the following law: The gravitational mass of a body is equal to its inertial mass.

(Einstein, Relativity, p. 65)

Relating Inertial and Gravitational Mass

- Choose a gravitational field with intensity g
- Whatever object we put in a given point in this field,it will always be accelerated by the same degree
- We already have: $\frac{M}{m} = \frac{a}{g}$
- We have in effect just said that when we hold g constant, a becomes a constant too
- So the ratio between M and m, i.e. $\frac{M}{m}$, must also remain constant
- In this case, we can appropriately choose our units for M and m so that $\frac{M}{m} = 1$, i.e $M = m$
  - This is just what we do: we measure **both** m and M in grammes

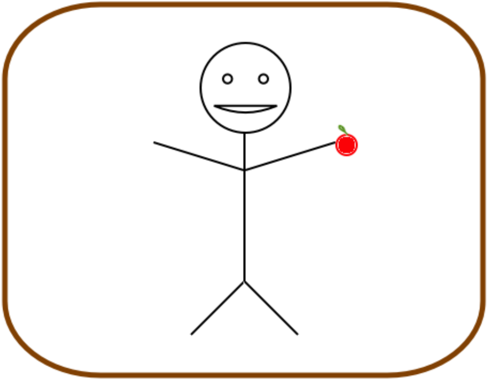
Relating Inertial and Gravitational Mass

> *It is true that this important law had hitherto been recorded in mechanics, but it had not been interpreted. A satisfactory interpretation can be obtained only if we recognize the following fact: The same quantity manifests itself according to circumstances as "inertia" or as "weight".*
>
> *(Einstein, Relativity, p. 65)*

The Equivalence Principle

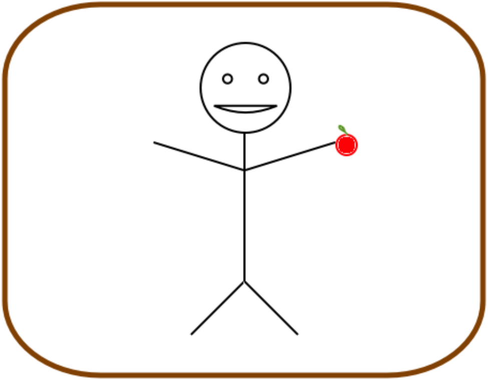
- GR solves this puzzle about mass through its key *Equivalence Principle*:
  - No experiment can distinguish between a frame of reference which is accelerating, and an inertial frame of reference which is in a gravitational field

A Though Experiment



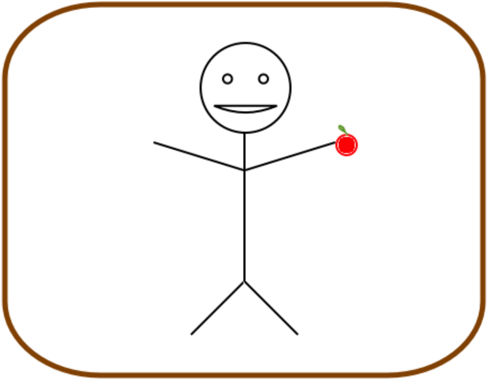
- Imagine an observer sitting in a sealed chest, which is floating in space somewhere far away from everything else

A Though Experiment



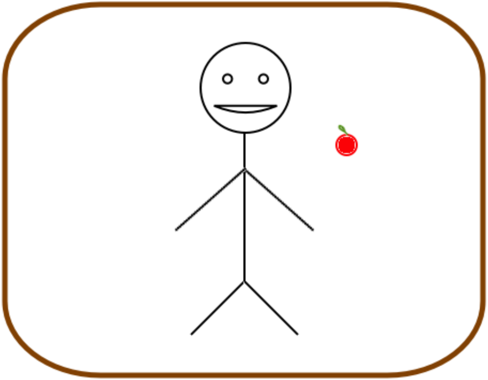
- For all intents and purposes, this chest is not in any gravitational fields, and is free falling

A Though Experiment



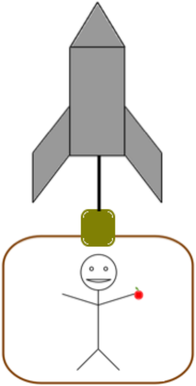
- The observer is free falling with the chest, and so will be floating inside the chest, 'weightless'

A Though Experiment



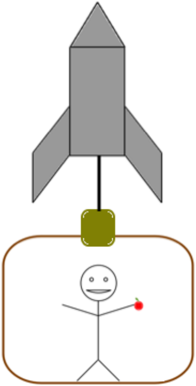
- If the observer lets go of an apple, it will stay exactly where it is, floating in the middle of the chest

A Though Experiment



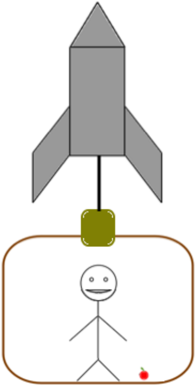
- Now imagine that we use a strong cable to attach the chest to a rocket ship, which is accelerating upwards, at a constant rate

A Though Experiment



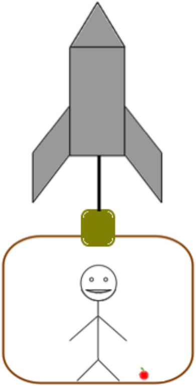
- The bottom of the chest will rush up to the observer, until they are standing on it

A Though Experiment



- Now if the observer drops an apple, it will eventually hit the bottom of the chest

A Though Experiment



- In other words, it will seem to the observer exactly like they are standing in a box in a gravitational field

A Though Experiment

Relying on his knowledge of the gravitational field [...], the man in the chest will thus come to the conclusion that he and the chest are in a gravitational field which is constant with regard to time. Of course he will be puzzled for a moment as to why the chest does not fall in this gravitational field. Just then, however, he discovers the hook in the middle of the lid of the chest and the rope which is attached to it, and he consequently comes to the conclusion that the chest is suspended at rest in the gravitational field.

A Though Experiment

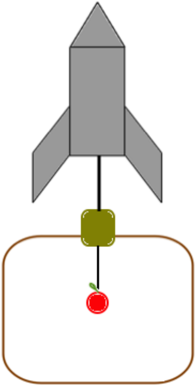
Ought we to smile at the man and say that he errs in his conclusion? I do not believe we ought to if we wish to remain consistent; we must rather admit that his mode of grasping the situation violates neither reason nor known mechanical laws. Even though it is being accelerated with respect to the "Galileian space" first considered, we can nevertheless regard the chest as being at rest. We have thus good grounds for extending the principle of relativity to include bodies of reference which are accelerated with respect to each other, and as a result we have gained a powerful argument for a generalized postulate of relativity.

(Einstein, Relativity, pp. 67–8)

Back to our Puzzle about Mass

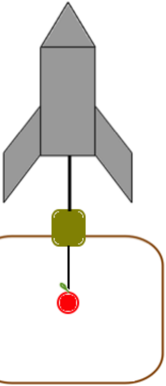
- This thought experiment thus supports the Equivalence Principle:
  - No experiment can distinguish between a frame of reference which is accelerating, and an inertial frame of reference which is in a gravitational field
- But how exactly does this principle solve our puzzle about masses?
- Recall that what we wanted was some sort of explanation of why inertial and gravitational mass of a body always equals its inertial mass

A Puzzle Solved



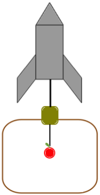
- Imagine that we tied one end of a rope to our apple, and the other end to the top of the chest

A Puzzle Solved



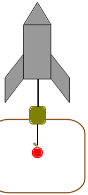
- There will be a measurable tension in the rope, which can be explained in one of two ways

A Puzzle Solved



- We can think of the chest as accelerating up
- We will say that the force on the chest is transmitted via the rope to the apple
- The tension in the rope is the force pulling on the apple to make it accelerate
- It is the **inertial** mass of the apple which determines the the magnitude of tension required to accelerate the apple

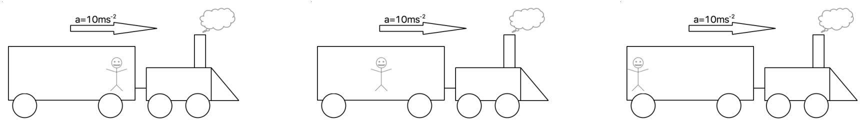
A Puzzle Solved



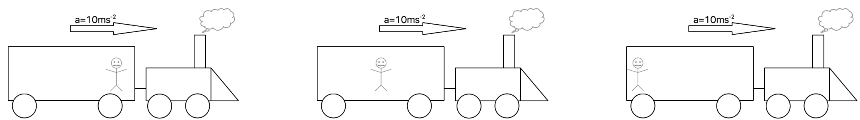
- Instead, we can think of the chest as being at rest in a gravitational field
- We will say that the apple experiences a gravitational force pulling it down
- The tension in the rope neutralises that gravitational force
- It is the **gravitational** mass of the apple which determines the magnitude of tension required to neutralise the gravitational force

An Obstacle Overcome

- But what does all of this have to do with the obstacle facing the General Relativity Principle?
  - The laws of nature are the same in **all** frames of reference
- Recall that our problem was with inertial forces: if we are in an accelerating train, then we can tell that we are because we will experience inertial effects



An Obstacle Overcome



- We can describe this situation in two ways
- We can say that the train is accelerating
  - The reason you hit the back of the carriage is that it "catches up" with you
- We can say that the train is at rest in a gravitational field
  - The reason you hit the back of the carriage is that the gravitational field "pulls" you

General Relativity

Three Principles of GR

- So far we have seen two principles of GR
- The General Relativity Principle:
  - The laws of nature are the same in **all** frames of reference
- The Equivalence Principle:
  - No experiment can distinguish between a frame of reference which is accelerating, and an inertial frame of reference which is in a gravitational field
- Now we need to add one more, *the Geodesic Principle*:
  - The natural state of motion for any object is free-fall; an object in free-fall follows a geodesic in spacetime

What does the Geodesic Principle Mean?

- *The Geodesic Principle*: the natural state of motion for any object is **free-fall**; an object in **free-fall** follows a geodesic in spacetime
- Essentially, this a modification of the principle that objects travel in straight lines unless a force acts on them
- An object is in **free-fall** if no forces are acting on it
  - What about gravity? As we will see, in GR gravity does not straightforwardly count as a force at all!

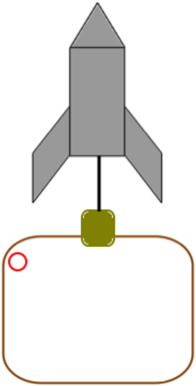
What does the Geodesic Principle Mean?

- *The Geodesic Principle*: the natural state of motion for any object is free-fall; an object in free-fall follows a **geodesic** in spacetime
- A geodesic is the shortest path between two points
- In flat, Euclidean geometry, the geodesics are just the straight lines
- However, in curved, non-Euclidean geometries, the geodesic diverge from straight lines
  - The more curved the geometry, the more the geodesics diverge from straight lines
- As we will see in a moment, the spacetime of GR is variably curved

Geodesic Paths and Light

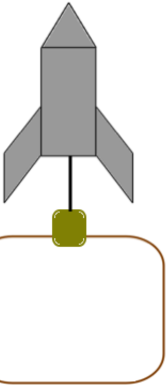
- Light (in a vacuum) always follows a geodesic path
- This is just a special instance of the Geodesic Principle
- Light (in a vacuum) is always in its natural state of motion, and so is always following a geodesic

Another Thought Experiment



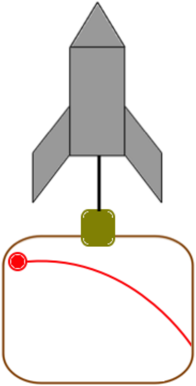
- Imagine that there is a flash bulb in one corner of a chest attached to a rocket accelerating up

Another Thought Experiment



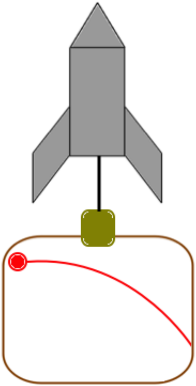
- The bulb goes off. What path will the light take through the chest?

Another Thought Experiment



- The light will take a curved path, hitting a lower point on the opposite wall of the chest

Another Thought Experiment

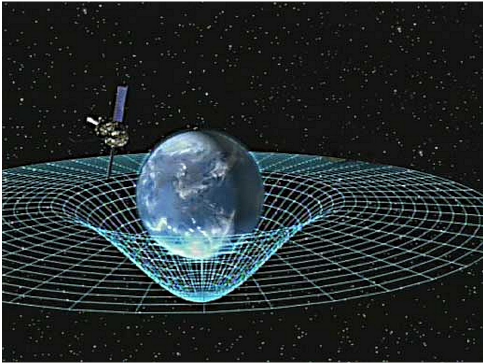


- So by the Equivalence Principle, it follows that a gravitational field would also bend the path of light

Bending Spacetime

- So gravitational fields bend the paths of light
- But we just said that light **always** follows a geodesic path
- So gravitational fields must bend spacetime itself
- This is the core of GR: mass-energy bends spacetime; the more mass-energy, the more the bending
- Einstein managed to calculate the exact degree to which mass-energy bends spacetime
- One of the crucial equations: $\mathbf{G} = 8\pi\mathbf{T}$
  - **G** is the Einstein tensor, which measures the curvature of spacetime
  - **T** is the mass-energy tensor, which measures the amount and distrubtion of mass-energy

General Relativity and Variable Curvature



Courtesy of NASA

- The spacetime of GR is variably curved (see Lecture 4)

General Relativity

General Relativity

- The General Relativity Principle:
  - The laws of nature are the same in **all** frames of reference
- The Equivalence Principle:
  - No experiment can distinguish between a frame of reference which is accelerating, and an inertial frame of reference which is in a gravitational field
- The Geodesic Principle:
  - The natural state of motion for any object is free-fall; an object in free-fall follows a geodesic in spacetime
- Mass-energy bends spacetime
  - $\mathbf{G} = 8\pi\mathbf{T}$

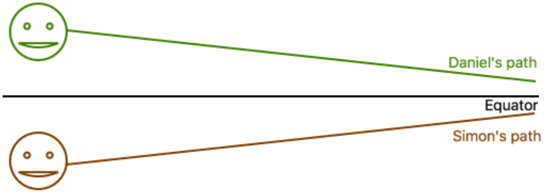
The GR Theory of Gravity

- According to GR, gravity **just is** the bending of spacetime
- Really, there is no force of attraction pulling objects together
- Massive objects bend spacetime, and this changes the paths that objects in free-fall follow
  - Objects in free-fall follow geodesics, and which paths count as geodesics is determined by the curvature of spacetime

Another Thought Experiment

- Daniel and Simon stand on either side of the equator, and try to walk in straight lines

Another Thought Experiment

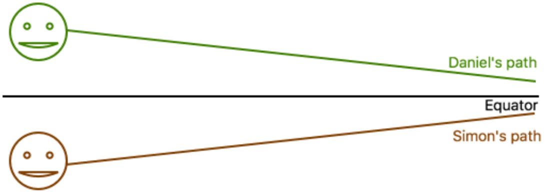


- As Simon and Daniel try to follow their straight paths, they find that they keep getting closer and closer together

Another Thought Experiment

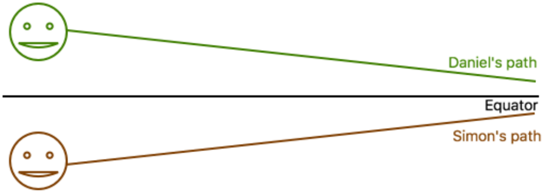
- They can stop this by putting a pole between them, but that produces a measurable tension in the pole

Another Thought Experiment



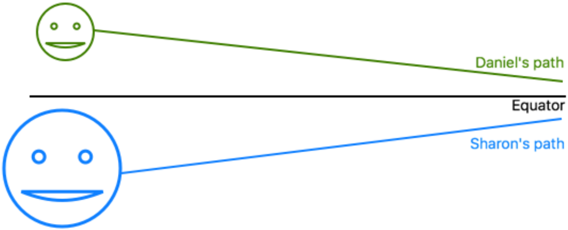
- Simon and Daniel conclude that there is a force of attraction pulling them together

Another Thought Experiment



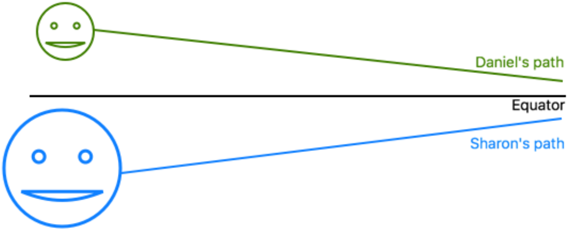
- To find out more about this force, Simon and Daniel start doing some experiments

Another Thought Experiment



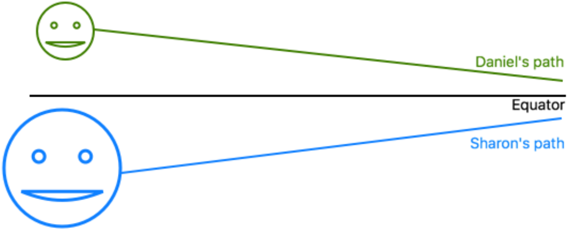
- They send Daniel and their much larger friend Sharon walking side by side

Another Thought Experiment



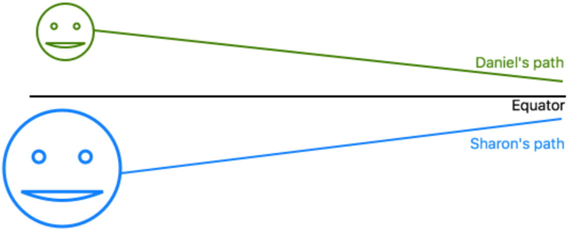
- They find that Daniel and Sharon get closer at exactly the same rate as Daniel and Simon

Another Thought Experiment



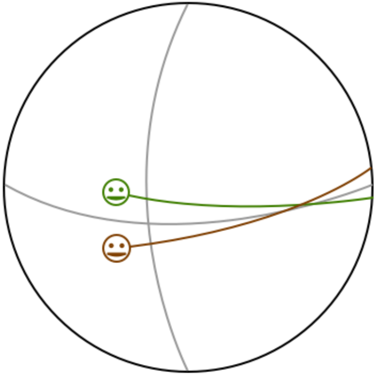
- They conclude that the attractive force between bodies must increase in proportion to the masses of those bodies

Another Thought Experiment



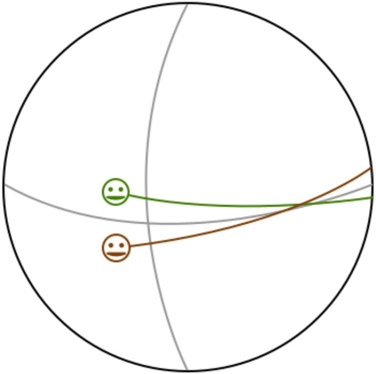
- They end up with something very much like Newton's theory of gravity!

Another Thought Experiment



- But really there is no attractive force between Daniel and Simon (or Sharon)

Another Thought Experiment



- They are on the surface of a sphere, and their “straight line paths” are really great circles around the sphere

Gravity is a Pseudo-Force

- According to GR, exactly the same is true of real gravity
- It seems as if there is a force of attraction pulling massive bodies together
- But there is no such force
- Really, massive bodies warp spacetime, and thereby change which paths count as geodesics

General Relativity

Absolutism and Substantivalism

- In Lecture 2, we looked at Newton's absolutist, substantivalist view of space and time
- Absolutism $\neq$ Substantivalism
- Absolutism means different things to different people, but as we have been using it, it is the doctrine that there are absolute facts about things like the following:
  - Which objects are at rest, which are moving
  - Which pairs of events are simultaneous
  - The spatial distance between two points
- Substantivalism is the idea that space is a kind of substance, a thing in its own right
- It is **very** hard to explain what substantivalism really amounts to
  - One suggestion: that we should take our apparent reference to and quantification over spacetime points at face value

GR as a Substantivalist Theory of Spacetime

- Clearly, the spacetime of GR is not absolute
  - All frames of reference (inertial or accelerating) can be treated as being at rest
  - In many cases, there is no absolute fact about whether two events are simultaneous
  - In many cases, there is no absolute fact about how far apart too points are
- But it is tempting to say that the spacetime is substantival
- Spacetime and matter **interact** with each other
- As John Wheeler puts it: "Matter tells space how to curve. Space tells matter how to move."

Introducing the Hole Argument

- So, GR seems to incorporate a substantivalist conception of spacetime
- *However*, it has also been argued that GR throws up a serious problem for substantivalism
- This is known as the **Hole Argument**, for reasons that will become clear shortly
- Einstein was the first to discover this argument while he was searching for the right formulation of GR
- It was resurrected and refurbished as an argument against spacetime substantivalism by John Earman and John Norton
- As we will see, the argument is in some ways similar to Leibniz's argument against Newton

Modelling the Universe

- A model of the universe is a triple, $\langle M, g, \mathbf{T} \rangle$
- M is a manifold
  - A manifold is a generalisation of Euclidean space
  - It can differ from Euclidean space in dimensionality, topology, geometry and so on
  - The manifolds of GR are 4-dimensional and are of variable curvature
- g is a metric tensor
  - A metric tensor tells us how to calculate the distance between points
  - In Euclidean space it simply reduces to Pythagoras' Theorem
  - In curved spaces things are much more complicated
- $\mathbf{T}$ is a stress-energy tensor
  - A stress-energy tensor describes the distribution of mass-energy throughout spacetime
  - Einstein's equations tell us how spacetime is curved as a result

Diffeomorphisms

- A diffeomorphism, d, is a special kind of function defined on the manifold, $d : M \to M$
- d is one-one (i.e. a bijection)
  - $\forall p \in M \exists q \in M (d(q) = p)$
  - $\forall p \in M \forall q \in M (p \neq q \to d(p) \neq d(q))$
- d is a continuous function
  - Roughly, if p, q, r... form a continuous path, then so do $d(p), d(q), d(r)$...
- The inverse of d, d^{-1}, is also continuous
  - $d^{-1}(p) = q \leftrightarrow d(q) = p$

From One Model to Another

- We can use diffeomorphisms to turn one model of the universe, $\langle M, g, \mathbf{T} \rangle$ into another, $\langle M, g', \mathbf{T}' \rangle$
  - $g'(p) = g(d^{-1}(p))$
  - $\mathbf{T}'(p) = \mathbf{T}(d^{-1}(p))$
  - In other words: the values of g' and $\mathbf{T}'$ at a given point p are the same as the values of g and $\mathbf{T}$ at the point which d sends to p
- It can be proven that $\langle M, g, \mathbf{T} \rangle$ is a GR model of the universe, then so is $\langle M, g', \mathbf{T}' \rangle$
- In fact, $\langle M, g, \mathbf{T} \rangle$ and $\langle M, g', \mathbf{T}' \rangle$ will be **qualitatively indistinguishable**

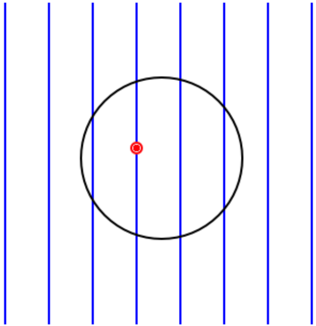
A Leibnizian Argument

- This should bring to mind Leibniz's arguments against substantivalism
- Leibniz complained that if substantivalism about space were true, then we could imagine numerically distinct yet qualitatively indistinguishable worlds
  - Compare the real world, and a world just like it except everything has moved 2m to my left
- Now, if spacetime substantivalism is true, then don't we also have to say that $\langle M, g, \mathbf{T} \rangle$ and $\langle M, g', \mathbf{T}' \rangle$ represent two numerically distinct but qualitatively indistinguishable worlds?

The Verification Principle

- As we saw back in Lecture 2, one of the weak points in Leibniz's argument is that it seems to rely on a form of **verificationism**
  - If there are no **empirically detectable** differences between universe A and B, then $A = B$
- If we left the argument against spacetime substantivalism just by complaining that $\langle M, g, \mathbf{T} \rangle$ and $\langle M, g', \mathbf{T}' \rangle$ represent two indistinguishable worlds, then we would also have to appeal to exactly the same kind of verificationism
- However, Earman and Norton develop the argument further, so that it doesn't have to rely on any kind of verificationism

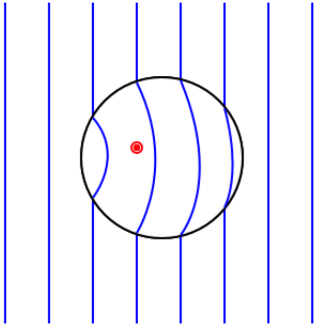
The Hole



$\langle M, g, \mathbf{T} \rangle$

- Consider a region within a universe, and call it "the hole"

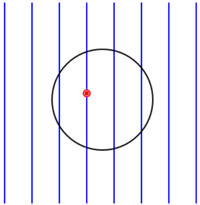
The Hole



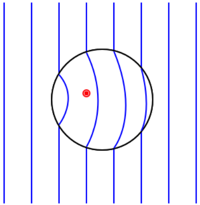
$\langle M, g', \mathbf{T}' \rangle$

- Now consider a diffeomorphism which is the identity map outside of the hole, but diverges within the hole

The Hole



$\langle M, g, \mathbf{T} \rangle$



$\langle M, g', \mathbf{T}' \rangle$

- These models represent indistinguishable universes
- Yet: in one universe objects following the blue trajectories pass through one set of points, and in the other universe they pass through another set of points
- Earman and Norton believe that this commits spacetime substantivalism to a radical form of **indeterminism**

Indeterminism

- **Determinism** can be characterised as follows:
  - The laws and initial conditions of the universe determine all later conditions of the universe
- However, the initial conditions of the universe along with the laws of nature (given to us by GR) do not tell us how an object will move through the hole
- Both $\langle M, g, \mathbf{T} \rangle$ and $\langle M, g', \mathbf{T}' \rangle$ satisfy the initial conditions and laws of nature
- This is a really weird form of indeterminism: no matter how much we know about the universe outside of the hole, and how small the hole is, we cannot say how things will move through it
- According to Earman and Norton, this is the price of spacetime substantivalism: it implies radical indeterminism

Is Indeterminism So Bad?

- Why is it a problem if spacetime substantivalism implies indeterminism?
- It would be bad news if we knew that determinism is true, but we don't
  - Although it's debatable, many people argue that Quantum Mechanics is inherently indeterministic
- **But:** the form of indeterminism implied by spacetime substantivalism is radical and bizarre
  - The hole could be anywhere, and could be as small as we like

Is Indeterminism So Bad?

- Moreover, Earman and Norton think that if we are going to sign up to indeterminism, then we should sign up to it for **physical** reasons, not philosophical ones
- But the indeterminism stemming from the Hole Argument is a consequence of a philosophical doctrine, spacetime substantivalism
- If we were relationists about spacetime, then we could say that $\langle M, g, \mathbf{T} \rangle$ and $\langle M, g', \mathbf{T}' \rangle$ represent the very same universe
  - Remember, they represent qualitatively indistinguishable universes

Is there any way out for the Substantivalists?

- $\langle M, g, \mathbf{T} \rangle$ is a model of the universe
- Which bit represents spacetime?
- Earman and Norton argue that it is the manifold, M
- But the manifold by itself lacks many spacetime properties, such as distance between points
- To include these properties, we need to take the manifold along with the metric tensor, g
- If we say that $\langle M, g \rangle$ is what represents spacetime, then the Hole Argument can't get going: if $g \neq g'$, then $\langle M, g \rangle$ and $\langle M, g' \rangle$ just represent different spacetimes
- However, we will discuss this way out of the Hole Argument further in the seminar

References

- Earman, J and Norton J (1987) 'What price space-time substantivalism? The Hole Story', *British Journal for the Philosophy of Science* 38: 515–25
- Einstein, A (1920) *Relativity: The Special and General Theory*

- **Helpful Reading**:
  - Dainton, B (2010) *Time and Space* (2nd ed), Chapters 20–21