The Philosophy of Physics
Lecture Six

The Conventionality of Simultaneity

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The Conventionality of Simultaneity

Einstein's Definition of Simultaneity

Measuring the Speed of Light

Reichenbach's Argument for Conventionality

Grünbaum's Argument for Conventionality

Malament's Argument against Conventionality

Einstein's Definition

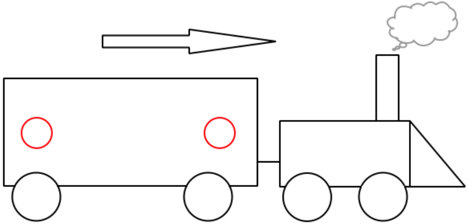
- Events A and B are simultaneous iff rays of light sent off from A and B would arrive at some point which is equidistant from A and B at the same time

- In Lecture 3, I used the **Light Postulate** to justify this definition

  The Light Postulate: the speed of light (in a vacuum) is a constant: c

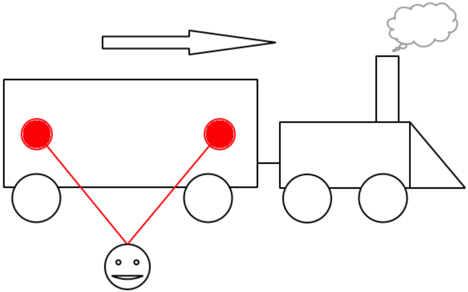
  - Suppose rays of light emitted from A and B meet at some point equidistant between them, C, at the same time
  - These rays of light travelled the same distance at the same speed
  - So they must have been emitted at the same time

The Relativity of Simultaneity



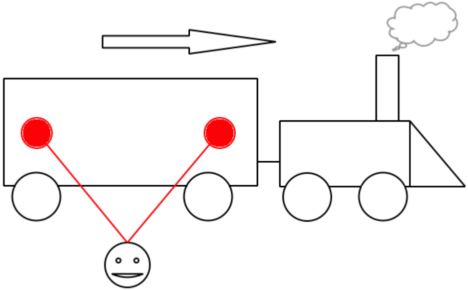
- There are two flash bulbs on a moving train

The Relativity of Simultaneity



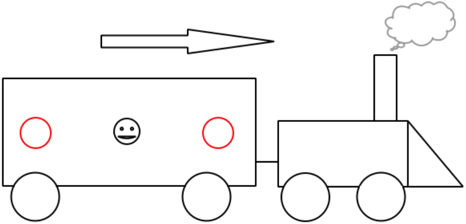
- They go off, and the rays reach an observer on the platform at the same time

The Relativity of Simultaneity



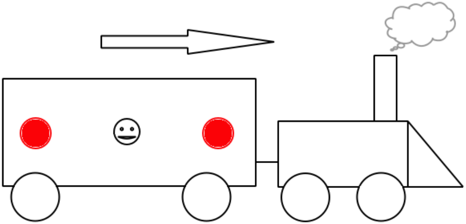
- The observer is equidistant between the two bulbs, and so from their perspective, they flashed at the same time

The Relativity of Simultaneity



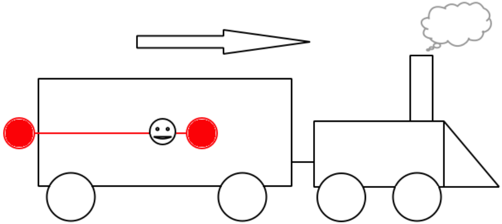
- But now imagine that there is someone inside the train

The Relativity of Simultaneity



- The bulbs go off, but the person in the train is moving towards the bulb on the right

The Relativity of Simultaneity



- The light from the bulb on the right will therefore reach the person in the train before the light from the bulb on the left

The Relativity of Simultaneity

- But now consider what the person on the train will see

The Relativity of Simultaneity

- Relative to this person's frame of reference, they remain constantly equidistant between the two bulbs

The Relativity of Simultaneity

- So they will say that the right bulb goes off **first**...

The Relativity of Simultaneity

- ...and the left bulb goes off **second**

The Relativity of Simultaneity

- So in SR, simultaneity is **relative** to a frame of reference
- Whether two space-like separated events count as simultaneous depends on which inertial frame we are using
- According to one frame, they will be simultaneous, but according to others they will not be

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The Conventionality of Simultaneity

- In this lecture we are going to look at the idea that simultaneity is not just **relative**, but **conventional** too
- In particular, the idea that whether two events count as simultaneous **according to a given inertial frame** is a matter of convention

The Conventionality of Simultaneity

- Suppose we are working with the frame of reference of this classroom
  - i.e. the frame of reference according to which this classroom is at rest
- And now suppose we want to ask whether two space-like separated events, A and B are simultaneous **according** to this frame of reference
- According to the Conventionality of Simultaneity, there is no objective, factual answer to this question
- Its all a matter of which conventions we adopt
- According to one convention they will be simultaneous, according to another they will not

How could Simultaneity Possibly be Conventional?

- There **is** an objective fact of the matter about whether rays of light emitted from A and B would reach C at the same time
- We can also assume that there is an objective fact of the matter about whether C is equidistant from A and B relative to any given frame of reference
- And the Light Postulate tells us that the speed of light is constant
- So where exactly is conventionality meant to creep in?

Two Versions of the Light Postulate

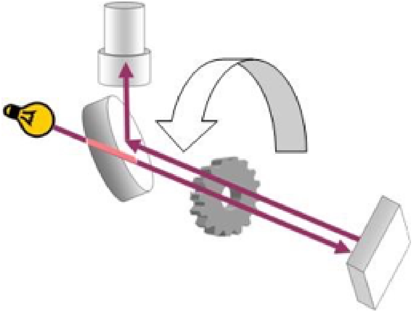
- The answer is: the Light Postulate
- In Lecture 3, I said that the Light Postulate is well confirmed, but that was a bit of a simplification
- Here can distinguish two versions of the Light Postulate:
  (1) **The one-way principle**: the speed of light is c in every direction
  (2) **The two-way principle**: On a round trip (in any direction), the average speed of light is c
    - A "round trip" is a trip from A to B and then back to A
- The version of the Light Postulate assumed in our earlier justification of Einstein's definition of simultaneity is the **one-way** principle
- But the version of the postulate that has actually been experimentally verified is the **two-way** principle

An Historical Claim

- All past determinations of the speed of light have been based on a round trip

Fizeau's Rotating Wheel

- All past determinations of the speed of light have been based on a round trip



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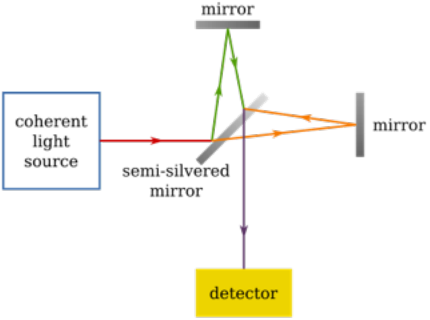
The Michelson-Morley Experiment

- All past determinations of the speed of light have been based on a round trip



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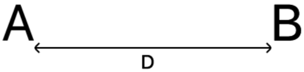
An “In Principle” Claim

- It is not just an accident that we haven’t ever measured the one-way velocity of light
- It is **in principle** impossible to measure the one-way velocity of light
  - Or at least, there are very good reasons for thinking that it is impossible

Why can't we Measure the One-Way Velocity of Light?

- Suppose we fired a beam of light from A to B, and we wanted to measure the speed of the light on this one-way journey

Why can't we Measure the One-Way Velocity of Light?

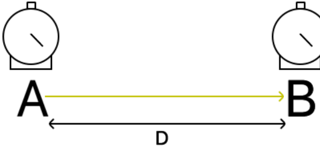


- Also suppose that we know the distance between A and B, call it D

Why can't we Measure the One-Way Velocity of Light?

- It would be easy to measure the oneway speed of light if we had two synchronised clocks, one at A and the other at B

Why can't we Measure the One-Way Velocity of Light?

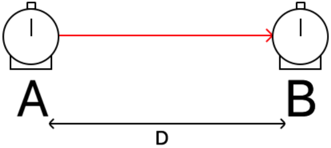


- All we would need to do is divide D by the time recorded on clock B at the moment the light reaches B

Why can't we Measure the One-Way Velocity of Light?

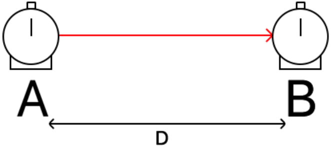
- But how would we synchronise these clocks?
- We could try by starting with the two clocks side by side, putting them on the same setting, and then taking them to A and B
- But we know that in the context of SR, there is no guarantee that the clocks will **stay** synchronised when they are moved to A and B
  - Remember the time dilation effects discussed in Lecture 3!

Why can't we Measure the One-Way Velocity of Light?



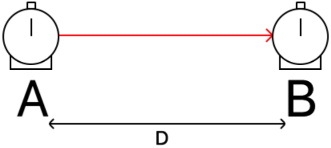
- Alternatively, we could send a signal from the clock at A to the clock at B

Why can't we Measure the One-Way Velocity of Light?



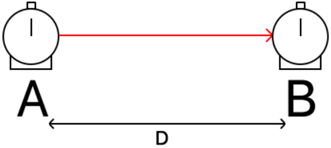
- This method would work perfectly if we could send infinitely fast signals

Why can't we Measure the One-Way Velocity of Light?



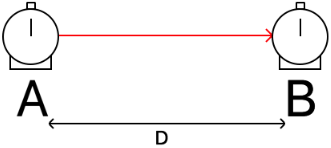
- If we sent the signal when the clock at A read 12:00, we would set B to 12:00 at the moment it received the signal

Why can't we Measure the One-Way Velocity of Light?



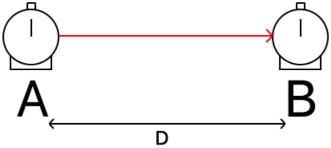
- But in SR, it is assumed that no signal can go faster than the speed of light

Why can't we Measure the One-Way Velocity of Light?



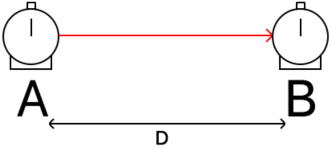
- So by the time the signal reaches the clock at B, some time will have ticked pass on the clock at A

Why can't we Measure the One-Way Velocity of Light?



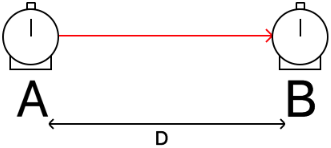
- We will need to compensate for that transit-time when setting the clock at B

Why can't we Measure the One-Way Velocity of Light?



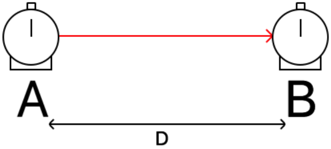
- But how much time will we need to compensate for?

Why can't we Measure the One-Way Velocity of Light?



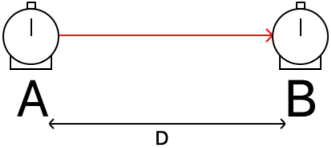
- That all depends on how fast the signal is travelling

Why can't we Measure the One-Way Velocity of Light?



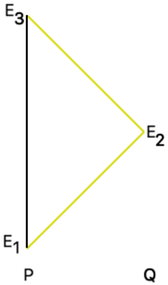
- But we can't measure **that** until we have synchronised our clocks at A and B!

Why can't we Measure the One-Way Velocity of Light?



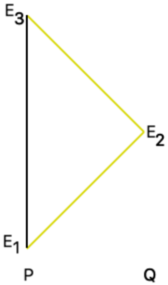
- Indeed, if we use a light signal, we are back where we started: trying to measure the one-way velocity of light

Simultaneity



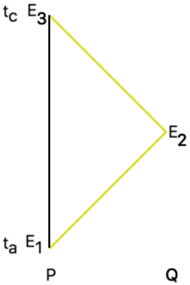
- Imagine that we are standing at point P, and fire off a ray of light (event E_1)

Simultaneity



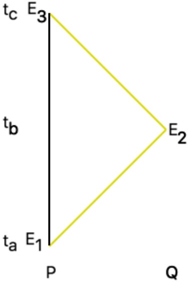
- The ray of light reflects off an object at point Q (event E_2), and then returns to P (event E_3)

Simultaneity



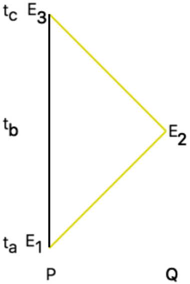
- We are standing at point P, and measure the time at E_1 (t_a) and E_3 (t_c)

Simultaneity



- Call the time that E_2 occurred according to our clock t_b

Simultaneity

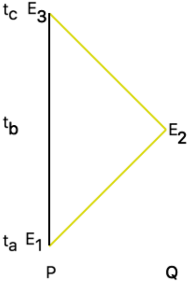


- How can we calculate t_b?

Simultaneity

- If we assume the one-way principle, then it is easy to calculate t_b
- The light would take exactly as long to go from Q to P as it took to go from P to Q
- So t_b would be exactly halfway between t_a and t_c
- $t_b = t_a + \frac{1}{2}(t_c - t_a)$

Simultaneity

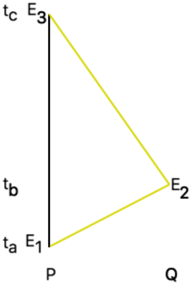


- $t_b = t_a + \frac{1}{2}(t_c - t_a)$

Simultaneity

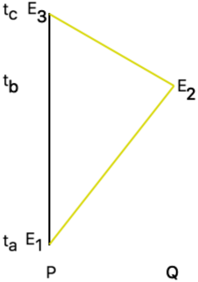
- But we have no empirical confirmation of the one-way principle
- All we have is the **two-way** principle
- And that is compatible with infinitely many values for t_b

Simultaneity



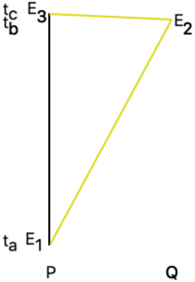
- $t_b = t_a + \frac{1}{4}(t_c - t_a)$

Simultaneity



- $t_b = t_a + \frac{3}{4}(t_c - t_a)$

Simultaneity

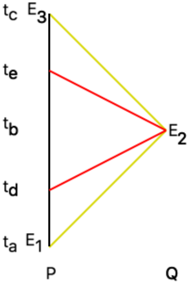


- $t_b = t_a + \frac{99}{100}(t_c - t_a)$

Simultaneity

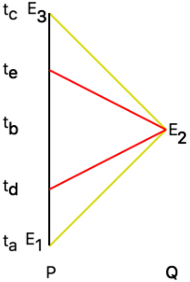
- $t_b = t_a + \epsilon(t_c - t_a)$
- All that our empirical observations require is that $0 < \epsilon < 1$
- If we assume the one-way principle, we get the "standard" value for ϵ: $\frac{1}{2}$
- But we can choose any other value without contradicting the two-way principle

Simultaneity



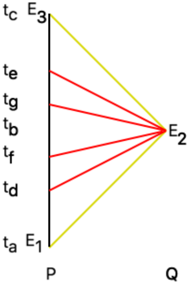
- Things would have been different if we could send signals faster than the speed of light

Simultaneity



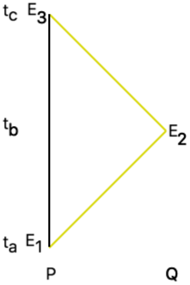
- Then we could narrow down the range of values that t_b could take

Simultaneity



- By sending faster and faster signals, we could pinpoint t_b as accurately as we liked

Simultaneity



- But light is the fastest signal, and so all we can ever say is that E_2 occurred at some time between t_a and t_c

Simultaneity

- We cannot know the one-way speed of light
- But we need to know the one-way speed of light to decide whether two space-like separated events are simultaneous, relative to some specified frame of reference
- As a result, we cannot know whether two space-like separated events are simultaneous, **even relative to a specified frame of reference**

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From Ignorance to Conventionality

- So far we have seen that we cannot **know** whether two space-like separated events are simultaneous, even relative to a specified frame of reference
- But this does not all by itself show that simultaneity is **conventional**, if this is taken to mean that there is no **fact of the matter** whether two events are simultaneous
- For all we have said so far, it may be that there **is** a fact of the matter, but we just cannot **know** it
- But that is a very unattractive position, and so it is very tempting to adopt conventionalism

Reichenbach on the Conventionality of Simultaneity

To determine the simultaneity of distant events we need to know a velocity, and to measure a velocity we require knowledge of the simultaneity of distant events. The occurrence of this circularity proves that simultaneity is not a matter of knowledge, but of a coordinative definition, since the logical circle shows that a knowledge of simultaneity is impossible in principle.

(Reichenbach, Philosophy of Space and Time, pp. 126f)

Einstein on the Conventionality of Simultaneity

- And in fact, Einstein himself said almost exactly the same thing
- He did not try to justify his definition of simultaneity with the one-way principle
- He simply laid down his definition as a conventional stipulation

Einstein on the Conventionality of Simultaneity

I feel constrained to raise the following objection: "Your definition [of simultaneity] would certainly be right, if I only knew that the light by means of which the observer at M perceives the lightning flashes travels along the length $A \to M$ with the same velocity as along the length $B \to M$. But an examination of this supposition would only be possible if we already had at our disposal the means of measuring time. It would thus appear as though we were moving here in a logical circle." After further consideration you cast a somewhat disdainful glance at me—and rightly so—and you declare:

Einstein on the Conventionality of Simultaneity

> *"I maintain my previous definition nevertheless, because in reality it assumes absolutely nothing about light. There is only one demand to be made of the definition of simultaneity, namely, that in every real case it must supply us with an empirical decision as to whether or not the conception that has to be defined is fulfilled. That my definition satisfies this demand is indisputable. That light requires the same time to traverse the path $A \to M$ as for the path $B \to M$ is in reality neither a supposition nor a hypothesis about the physical nature of light, but a stipulation which I can make of my own freewill in order to arrive at a definition of simultaneity."*

(Einstein, Relativity, pp. 22–3)

Reichenbach's Argument

- What do we need to assume to jump from this ignorance to conventionalism?
- *Verificationism*: A statement has a truth-value iff it is empirically testable
- It is impossible to empirically test the one-way principle
- Thus there is no fact of the matter about the one-way speed of light
  - There is no fact of the matter about $p =_{df}$ the sentence 'p' does not have a truth-value, true or false
- Therefore, the choice to endorse the one-way principle ($\epsilon = \frac{1}{2}$) is a convention, not a description of reality

Responding to Reichenbach

- Find a way to measure the one-way velocity of light
  - But we already saw that there are good reasons for thinking that this is impossible in principle
- Reject verificationism
- Appeal to some other principle, e.g. simplicity, to provide empirical support for our assumptions about the one-way speed of light
  - The assumption that the one-way speed = the two-way speed seems in some ways like the simplest. But why should that motivate us? Is simplicity a sign of truth?
- Link the one-way principle to something else in physics that seems well established
  - e.g. space is isotropic (i.e. the same in all directions). But where does the isotropy of space come from? Is it just a convention too?

Verificationism

- The obvious weak link is **verificationism**
- Verificationism is a very restrictive theory of meaning, and has lots of apparent counterexamples:
  - Logical claims: All vixens are vixens
  - Analytic claims: All vixens are female foxes
  - Ethical claims: murder is wrong
  - Metaphysical claims: the external world exists
  - Religious claims: God is good

Verificationism

- Verificationists came up with theories dealing with all of these claims
  - Logical and analytic truths(/falsehoods) are conventional claims, made true(/false) by the conventions governing our language
  - Ethical claims are expressions of our attitudes to various acts (Boo to murder!)
  - Metaphysical claims are sheer nonsense, which should be rejected as meaningless
  - Religious claims are either expressions our attitudes to the world (Yay to the world!), or else are nonsensical metaphysical claims
- But these are all very controversial theories!

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Conventionality without Verificationism

- It would be nice, then, if we could come up with an argument for the conventionality of simultaneity **without** assuming verificationism
- Grünbaum has presented just such an argument
- Instead of relying on verificationism, Grünbaum relies on some assumptions about time

Reichenbach's Causal Theory of Time

- The Causal Theory of Time:
  - The **temporal order** of events is reducible to the **causal** relations between events
- Reichenbach's Causal Theory of Time:
  - E_1 is **before** E_2 iff E_1 can causally affect E_2
  - E_1 is **simultaneous** with E_2 iff E_1 is not before E_2 and E_2 is not before E_1

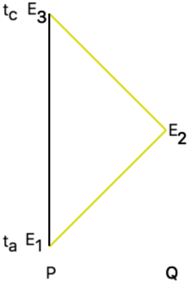
Reichenbach's Causal Theory of Time

- Reichenbach's version of the causal theory is intuitive
  - Causes precede their effects
- But it turns out to be tricky to decide which of the two events in a causal relationship is the cause, and which the effect?
  - E_1 and E_2 are causally related
  - Does E_1 cause E_2, or the other way around?
- It would be easy to answer this question if we knew which came first, E_1 or E_2
  - Causes precede their effects, and so whichever came first was the cause
- But in Reichenbach's theory, we are meant to use the causal relationship between E_1 and E_2 to decide which came first!

Grünbaum's Causal Theory of Time

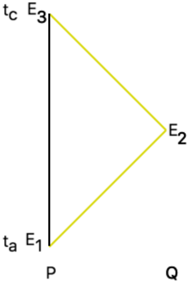
- Grünbaum's Causal Theory of Time:
  - There is a temporal separation between E_1 and E_2 iff E_1 and E_2 are causally connectible
    - To say that there is a temporal separation between E_1 and E_2 is to say that either E_1 is before E_2, or E_2 is before E_1
  - E_1 and E_2 are simultaneous iff it is not the case that there is a temporal separation between E_1 and E_2
- More precisely, Grünbaum calls this kind of simultaneity **topological simultaneity**
- Events E_1 and E_2 are topologically simultaneous iff they are space-like separated

Topological Simultaneity



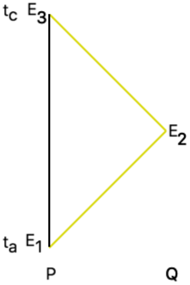
- E_1 and E_2 are causally connected, so they are temporally separated

Topological Simultaneity



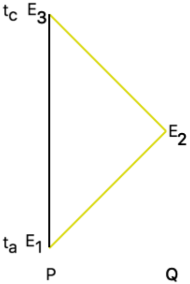
- E_2 and E_3 are causally connected, so they are temporally separated

Topological Simultaneity



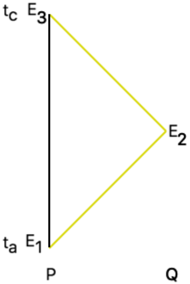
- E_2 is space-like separated from all the events lying on the path from E_1 to E_3

Topological Simultaneity



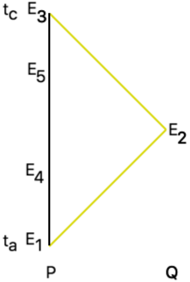
- So E_2 is topologically simultaneous with **all** those events

Topological Simultaneity



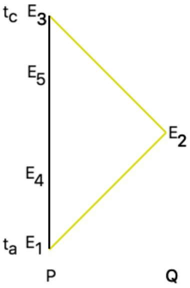
- Topological simultaneity is an odd kind of simultaneity: it is not transitive

Topological Simultaneity



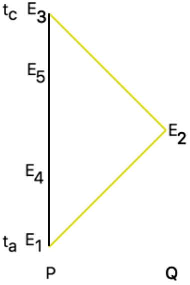
- E_4 is topologically simultaneous with E_2

Topological Simultaneity



- E_2 is topologically simultaneous with E_5

Topological Simultaneity



- But E_4 is not topologically simultaneous with E_5: E_4 and E_5 are causally connectible, and so temporally separated

Metrical Simultaneity

- But for many physical purposes, we need a notion of simultaneity which is transitive (and symmetric, and reflexive)
- So we need another notion of simultaneity, in addition to topological simultaneity, called **metrical simultaneity**
- But nothing in objective reality forces us to pick a particular event between E_1 and E_3 as being metrically simultaneous with E_2
  - All that objective reality supplies us with is topological simultaneity
- Which event we choose to treat as being metrically simultaneous with E_2 is a matter of convention
  - Our choice will be equivalent to a choice about the one-way velocities of light

The Conventionality of Simultaneity

Einstein's Definition of Simultaneity

Measuring the Speed of Light

Reichenbach's Argument for Conventionality

Grünbaum's Argument for Conventionality

Malament's Argument against Conventionality

Malament's Result

- If we wanted to block Grünbaum's argument for the conventionality of simultaneity, we could reject the causal theory of time
- **Or**, we could try showing that even given the causal theory of time, there is a way of Privileging a unique metrical simultaneity relation
  - In other words, we will show that **causal connectability relations** privilege a unique metrical simultaneity relation
- This is just what Malament did in 1977

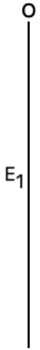
Step One: Defining the Standard Simultaneity Relation

- Malament's first step was to show that we could define the **standard simultaneity** relation (where light travels in the same speed in all directions) in causal terms
- The first thing we need to is introduce a frame of reference, since simultaneity is only ever defined relative to a frame of reference
- So let's focus on the frame of reference of some stationary observer, and call his path through spacetime O

Step One: Defining the Standard Simultaneity Relation

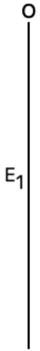
- O is the path of our stationary observer through spacetime

Step One: Defining the Standard Simultaneity Relation



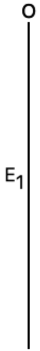
- Now consider some event E_1 on O

Step One: Defining the Standard Simultaneity Relation



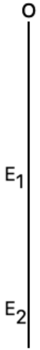
- We want to find a way of defining the standard hyperplane of simultaneity, s, for E_1

Step One: Defining the Standard Simultaneity Relation



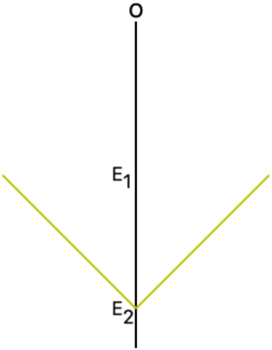
- *s* is the hyperplane of simultaneity we get if we say that the speed of light is constant in all directions

Step One: Defining the Standard Simultaneity Relation



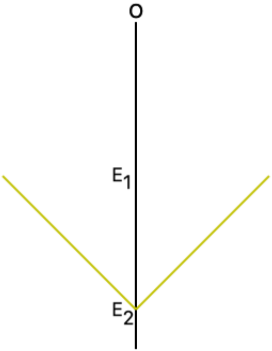
- Choose any event on O before E_1, call it E_2

Step One: Defining the Standard Simultaneity Relation



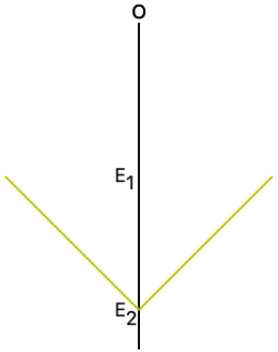
- Consider the possible light paths from E_2

Step One: Defining the Standard Simultaneity Relation



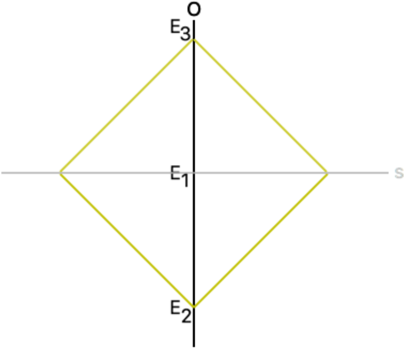
- These lines must eventually intersect each possible simultaneity hyperplane for E_1

Step One: Defining the Standard Simultaneity Relation



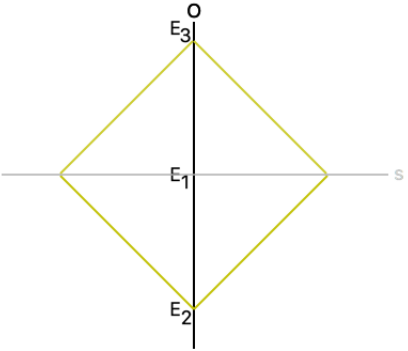
- We can now single out the standard simultaneity hyperplane, *s*, like this:

Step One: Defining the Standard Simultaneity Relation



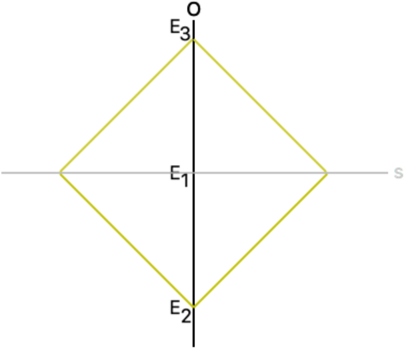
- If we reflect the light paths back towards O when they intersect s, then they will all arrive back on O at the same event, E_3

Step One: Defining the Standard Simultaneity Relation



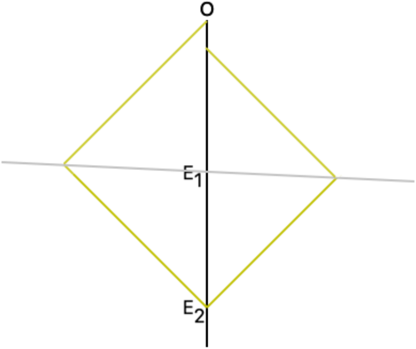
- s is defined as the one and only simultaneity hyperplane for E_1 which has this property

Step One: Defining the Standard Simultaneity Relation



- It is also immediately clear from this definition that s must be orthogonal to O

Step One: Defining the Standard Simultaneity Relation



- It is also immediately clear from this definition that s must be orthogonal to O

Step Two: Privileging the Standard Simultaneity Relation

- Step One was just showing that we could define the standard simultaneity relation in causal terms
- That was the easy step!
- Step Two is to show that the causal relations privilege the standard simultaneity relation

Step Two: Privileging the Standard Simultaneity Relation

- Malament proved that the standard simultaneity relation is the only relation defined in terms of O and causal connectability which meets the following two conditions:
  (i) The relation is not **trivial**
    - The relation does not relate every event to every other event
    - The relation does not fail to relate events on O to events not on O

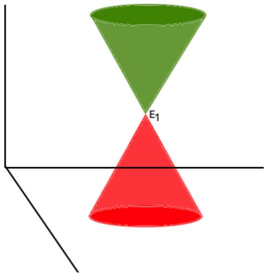
  (ii) The relation is an equivalence relation
    - i.e. the relation is reflexive, symmetric and transitive

Step Two: Privileging the Standard Simultaneity Relation

- Suppose that some relation R is definable in terms of O and causal connectability
- R will be invariant under all transformations that preserve the positioning of O and all the causal connectability relations
  - In other words: if f is such a function, then $R(E_1, E_2) \rightarrow R(f(E_1), f(E_2))$

Causal connectability and the Light Cone Structure

- In SR, the causal connectability structure is just the light cone structure



- E_1 and E_2 are causally connectible iff E_2 lies in or on one of E_1's light cones

Step Two: Privileging the Standard Simultaneity Relation

- If R is definable in terms of O and causal connectability, then R will be invariant under all transformations that preserve the positioning of O and **the light cone structure**
- Malament proved that the standard simultaneity relation is the **only** non-trivial equivalence relation which is invariant under all such transformations

The Transformations

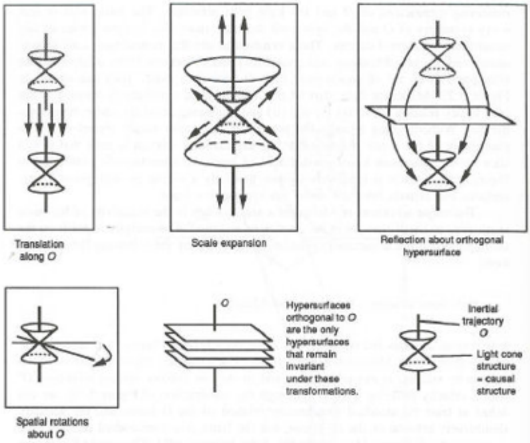


Figure 5.22 Symmetries of the light cone structure of a Minkowski spacetime and an inertial trajectory O.

Courtesy of John Norton (1992), p. 226

Step Two: Privileging the Standard Simultaneity Relation

- We will not go through Malament's proof, but the core idea is this
- The reason that the standard simultaneity relation is the only non-trivial equivalence relation which is preserved under these transformations is that it defines the orthogonal simultaneity hyperplane
- Any hyperplane which is **not** orthogonal will fail to be invariant under at least one of the above transformations

Has Malament Proven that Simultaneity is not Conventional?

- That partly depends on whether or not we think that anything conventional has crept into Malament's background assumptions
- For example, Malament clearly assumes that simultaneity be an equivalence relation
- Perhaps that is a conventional assumption?
- Something to discuss in the seminar!

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- Grünbaum, A (1973) *Philosophical Problems of Space and Time*, 2nd enlarged edition (Dordrecth/Boston: D Reidel)
- Malament, D (1977) 'Causal Theories of Time and the Conventionality of Simultaneity', *Nous* 11: 293–300
- Norton, J (1992) 'The Philosophy of Space and Time', Chapter 5 in Salmon et al (eds) *Introduction to the Philosophy of Sicence*, pp. 222–6
- Reichenbach, H (1957), *The Philosophy of Space and Time*, esp. pp. 123–35 (§§19–20)