The Philosophy of Physics
Lecture Eight

EPR and the Bell Results

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EPR and the Bell Results

Einstein and Quantum Mechanics

- Einstein had a famously difficult relationship with QM
- Einstein's own work on the photoelectric effect was a crucial step towards QM
- But Einstein thought that QM was importantly **incomplete**
- This does not mean that he thought that QM was in any way **inaccurate**
  - Einstein wasn't saying that any of the predictions of QM were false
- Einstein simply thought that the formalism of QM was not provide a *complete* description of the quantum world

What does 'Complete' Mean?

- When a quantum system is in a state which is not an eigenstate of a given property P, QM refuses to attribute a definite P-value to that state
  - e.g. if an electron has a definite momentum, then QM does not attribute a definite position to that electron
- But Einstein was convinced that electrons always have definite positions, and momentums, and spins, etc...
- So Einstein thought that quantum systems had properties that were not reflected in the formalism of QM
- This is the sense in which Einstein thought that QM was incomplete

Einstein's Definition of 'Complete'

> *Whatever the meaning assigned to the term complete, the following requirement for a complete theory seems to be a necessary one: every element of the physical reality must have a counterpart in the physical theory.*
>
> *(EPR p. 777)*

Completeness and the Copenhagen Interpretation

- Einstein's insistence that QM is incomplete goes against the Copenhagen Interpretation
- According to Copenhagen, QM **is** complete
- If QM does not attribute a definite position to an electron, then that electron has no definite position
- If we then measure the position of that electron, that act of measurement somehow **makes** that electron have a definite position
  - Exactly how is a deep problem, and is one aspect of the **measurement problem**
- According to Copenhagen, QM leaves nothing out

A Philosophical Debate?

- You might have thought that the kind of debate between Einstein and Bohr (or our imaginary Copenhagen) is inherently **philosophical**, rather than narrowly physical
- After all, it involves arguing over physical **reality**, and the word ‘reality’ has a suspiciously metaphysical ring to it
- But amazingly, Einstein (along with Podolsky and Rosen, jointly known as **EPR**) managed to turn it into a physically tractable debate

A Sufficient Condition for Reality

- The first step is to offer a sufficient condition for being real:

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*
>
> *(EPR p. 777)*

- Importantly, this is only a **sufficient** condition, not a **necessary** one
  - If something meets this condition then it is real, but if it fails this condition then that does not automatically mean it isn't real

EPR's Strategy

- EPR want to use this criterion to argue that quantum particles always have a full set of determinate properties, **even though the QM formalism does not attribute such a set of properties to them**
- If their argument works, then QM is incomplete, because there are aspects of reality which are not reflected in its formalism

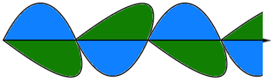
EPR and the Bell Results

A Reconstruction

- In what follows, I am going to offer a **simplified reconstruction** of the EPR argument
- This reconstruction is taken from chapter 1 of Maudlin's *Quantum Non-Locality and Relativity*
  - I **highly** recommend this chapter for everything to do with this week's lecture
- But before I can present this reconstruction, I need to introduce you to the idea of **polarisation**

Light as an Electromagnetic Wave

- Forget for a moment that light can be quantised into photons, and just think of it as an electromagnetic wave
- Electromagnetic waves are disturbances of electric and magnetic fields



- The electric and magnetic fields are both perpendicular to the direction in which the wave is travelling
- And they are perpendicular to each other, too

Polarised Light

- Although the electric field must always be perpendicular to the magnetic field and the direction the wave is travelling, it can still change its direction as the wave propagates
  - For example, it could rotate around the direction in which the light is travelling
- But in some special cases, the electric field does not change its direction
  - In other words: it just keeps oscillating on the same plane
- This is called **plane polarised light**

Polarising Filters

- Normal sunlight is not polarised, but we can send it through a **polarising filter**, which we can set at any angle
- It decomposes incoming electromagnetic waves into two components:
  - A component which is polarised in the direction of the filter's polarisation
  - A component which is polarised perpendicularly to the filter's polarisation
- The polarising filter then absorbs the second component, and only allows the first to pass
- As a result, light passing through a polarising filter tends to become less intense (bright)

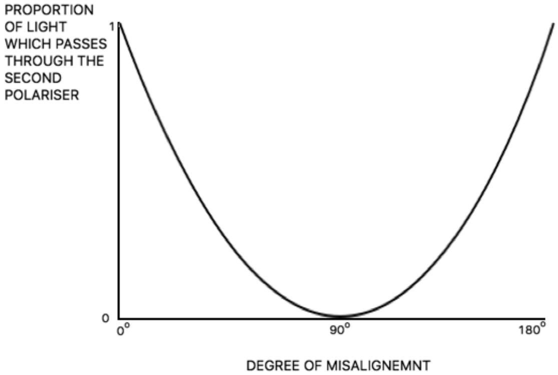
Two Polarising Filters

- Now imagine that you had two polarising filters arranged in a series: light first goes through one, then the other
- Now set the angle of the first polarising filter at θ
- Lots of the light that initially hit the first polariser would be absorbed, but all the light that passed would now be polarised to θ
- How much of this now polarised light will pass through the second filter?
- That all depends on the degree of **misalignment** between the two filters

Two Polarising Filters

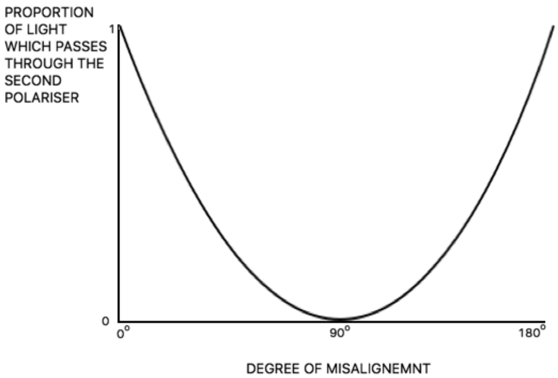
- If they are perfectly aligned, so that the second filter is also set at θ, then all of the polarised light will pass through
- But if the second filter is set to $\theta + 90^\circ$, then all of the polarised light will be absorbed
- And if the second filter is set to an angle between θ and $\theta + 90^d egree$, then the amount of light which will get through is described by the following graph

Two Polarising Filters



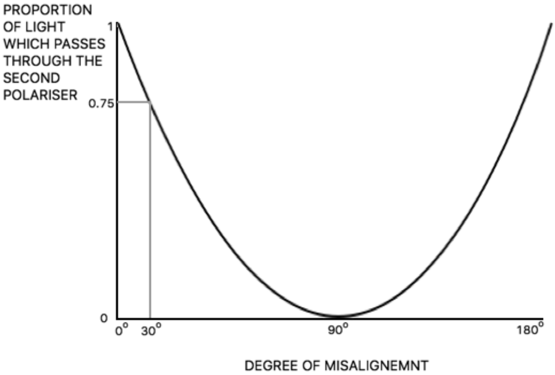
- At 0° misalignment, all of the light passes through the second filter

Two Polarising Filters



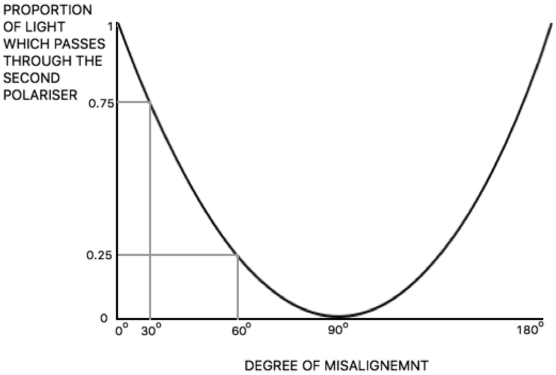
- At 90° misalignment, none of the light passes through the second filter

Two Polarising Filters



- At 30° misalignment, 0.75 of the light passes through the second filter

Two Polarising Filters



- At 60° misalignment, 0.25 of the light passes through the second filter

Light as a Beam of Photons

- Now let's remember again that light can be thought of as a beam of photons
- Typically, polarising filters absorb at least some of the energy from the light that hits them
  - The only exception is if the light happened to be polarised in the direction of the filter
- So you might have thought that when light passes through a filter, each photon looses a bit of its energy

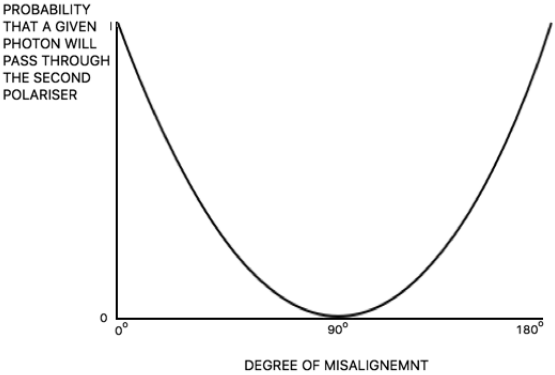
Light as a Beam of Photons

- But in fact, a photon has exactly the same energy before and after it passes through a polariser
  - Remember, the energy of a photon is proportional to the frequency of the associated wave, and polarisers do not affect frequency
- Instead, a polarising filter only lets **some** of the photons in a beam of light pass through it
- The rest are absorbed by the filter, which is why the beam typically loses some energy as it passes through the filter

Two Polarising Filters (Again)

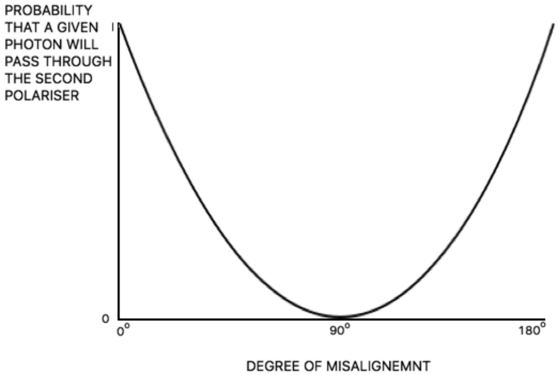
- So imagine again our set up of two polarising filters in a series
- When we were thinking of light as a wave, we said:
  - The **proportion of light** which is absorbed by the second filter depends on the degree of misalignment between the two filters
- Now that we are thinking of light as a beam of photons, we can say:
  - The **probability that a given photon** will be absorbed by the second filter depends on the degree of misalignment between the two filters

Two Polarising Filters (Again)



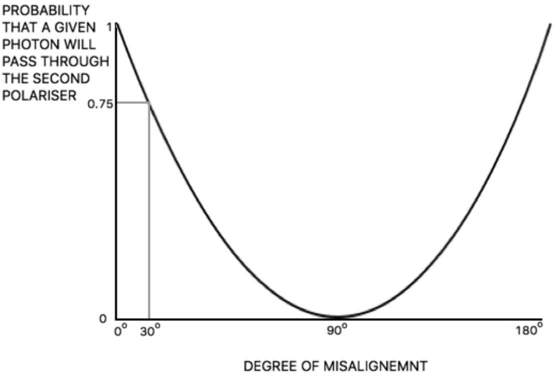
- At 0° misalignment, a given photon is certain to pass through the second filter

Two Polarising Filters (Again)



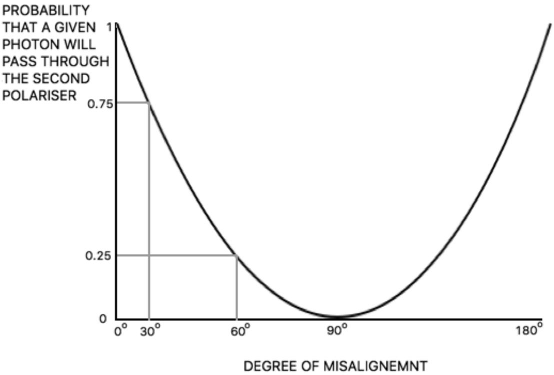
- At 90° misalignment, a given photon is certain not to pass through the second filter

Two Polarising Filters (Again)



- At 30° misalignment, the probability of a given photon passing through the second filter is 0.75

Two Polarising Filters (Again)



- At 60° misalignment, the probability of a given photon passing through the second filter is 0.25

EPR and the Bell Results

Entanglement

- According to QM, two quantum particles can become **entangled**
- This means that the outcome of certain measurements done on these particles are guaranteed to be related in certain ways
  - Sometimes we are guaranteed that the measurements will have the **same** result
  - Sometimes we are guaranteed that the measurements will have **different** results

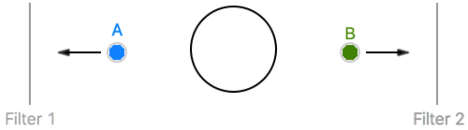
Why Entanglement Happens

- When we are dealing with the state of a system involving two particles, we can sometimes **separate** it into a state of one particle and a state of the other
- But if the particles are interacting in certain ways, we cannot do that: the state is an **inseperable** unity
- In such a system, the behaviour of both particles is governed by the **same** wave function
- Measuring **either** particle is enough to collapse that wave function for **both** particles

Entangling Photons

- If you fire a laser tuned to a certain frequency at calcium vapour, the vapour fluoresces
- Each atom emits two photons, which are fired off in opposite directions
- There is no systematic polarisation of these photons
  - No matter what angle θ we choose, half the photons fired will pass through a polarising filter set to angle θ
- However, QM predicts that each pair of emitted photons are guaranteed to be polarised in the same way

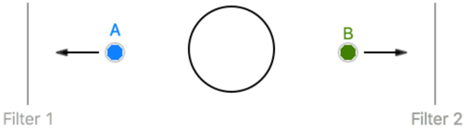
Entangling Photons



- If the two filters are aligned to the same angle and A passes through Filter 1, then QM predicts that B is certain to pass through Filter 2

- If the degree of misalignment is 90° and A passes through Filter 1, then QM predicts that B is certain not to pass through Filter 2

Entangling Photons



- If the degree of misalignment is 30° and A passes through Filter 1, then QM predicts that B has a 0.75 chance of passing through Filter 2

- If the degree of misalignment is 60° and A passes through Filter 1, then QM predicts that B has a 0.25 chance of passing through Filter 2

EPR and the Bell Results

The EPR Argument: The Assumptions

- The EPR argument starts with two assumptions:
  (1) **QM is accurate:** photons behave as QM predicts
  (2) **Locality:** no causal influence can propagate faster than the speed of light

- (1) is a good starting point, because EPR want to convince people who accept that QM is an accurate theory

- And (2) seems fine, because it is often said that Locality follows from Einstein's theories of Relativity

The EPR Argument

- When the pair of photons are emitted, QM attributes no definite polarisation to either photon
- We can measure the polarisation of A by passing it through a polarising filter
- At this point QM tells us that we can predict with certainty what the polarisation of B is: it is polarised in the same way as A
- So if **QM is accurate**, then we can predict the polarisation of B with certainty by measuring the polarisation of A

The EPR Argument

- By **Locality**, we can arrange things to ensure that B was not disturbed when we measured the polarisation of A
- We can arrange the filters so that the event of A arriving at Filter 1 is **space-like separated** from the event of B arriving at Filter 2
- So by Einstein's earlier criterion of reality, B must have had a definite polarisation all along

Einstein's Sufficient Condition for Reality

If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

(EPR p. 777)

The EPR Argument: Conclusion

- EPR took this as proof that QM is **incomplete**
- When the two photons are emitted, QM assigns no definite polarisation to B
- But B must have had such a polarisation, because we can predict with certainty what polarisation B will have without disturbing B
- We can do this by measuring A: given Locality, our measuring A cannot affect B in any way
- Thus QM is missing some **hidden variables**

Bohr's Response to EPR

> *Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of* **an influence in the very conditions which define the possible types of predictions regarding the future behaviour of the system**.
>
> *(Bohr 1935, p. 700)*

- Clearly, Bohr is trying to make space for the idea that **some kind of influence** is not bound by Locality, but it is hard to understand exactly what is intended

Bohr's Response to EPR

- Fortunately, however, we do not need to try to figure out what Bohr had in mind
- Amazingly, in 1964 John Stewart Bell published a paper which showed that Einstein's and Bohr's competing pictures of reality resulted in different **empirical** predictions
- Einstein's claim that QM was incomplete could be put to the test!

EPR and the Bell Results

A Fun Game for You and a Friend!

- Following Maudlin (2011: 13–7), I will introduce Bell's result by imagining a game for you and a partner
- In each round, you and your partner will be taken to separate rooms, and you will both be asked one of three questions: 0? 30? or 60?
- In response to the question you are asked, you must either answer 'Passed' or 'Absorbed'
- You and your partner may be asked the same question or different ones, and you do not know which question either of you will be asked
- There will be many rounds of this game (let's say 1,000)

A Fun Game for You and a Friend!

- The aim of the game is to re-create the behaviour of the photons in the scenario described earlier

- More precisely, once you have played many rounds, you want the following statistics to come out true:

  - If you and your friend are asked the same question, then you always give the same answer
  - When the questions differ by 30 (i.e. one of you is asked 0? and the other is asked 30?, or one of you is asked 30? and the other is asked 60?) then you give the same answer 0.75 of the time
  - When the questions differ by 60 (i.e. one of you is asked 0? and the other is asked 60?) then you give the same answer 0.25 of the time

A Fun Game for You and a Friend!

- In each round you will have a chance to confer before you go into your separate rooms, and during those conferences you can change your strategies in any ways you like
- **So what strategy would let you and your partner win this game?**

Stochastic Strategies are no Good

- Clearly, it wouldn't be a good idea for you and your partner just to answer your questions randomly (say by flipping a coin)
- That would never guarantee that you would always give the same answer if you were asked the same question
- You and your partner need to settle on a strategy which is deterministic in the following sense:
  - You both know exactly how the other will answer each possible question
- Of course, you might want to introduce some sort of randomisation in how you choose this strategy, but the strategy **itself** is deterministic

The Possible Strategies

- There are 8 possible deterministic strategies for answering the series of questions $\langle 0?, 30?, 60? \rangle$:

$$
\begin{array}{llll}
(1) & \langle P, P, P \rangle & (2) & \langle A, A, A \rangle \\
(3) & \langle A, P, P \rangle & (4) & \langle P, A, A \rangle \\
(5) & \langle P, A, P \rangle & (6) & \langle A, P, A \rangle \\
(7) & \langle P, P, A \rangle & (8) & \langle A, A, P \rangle
\end{array}
$$

- In order to guarantee that you always give the same answer if you are asked the same question, both of you must pick the same strategy as each other in each round

Halving the Field

(1)	$\langle P, P, P \rangle$	(2)	$\langle A, A, A \rangle$	(A)
(3)	$\langle A, P, P \rangle$	(4)	$\langle P, A, A \rangle$	(B)
(5)	$\langle P, A, P \rangle$	(6)	$\langle A, P, A \rangle$	(C)
(7)	$\langle P, P, A \rangle$	(8)	$\langle A, A, P \rangle$	(D)

- Since we only care if you and your partner give the same answer, we can simplify the space of options a bit
- We can regard each line on this table as just **one** strategy:
  - (1) and (2) are both versions of strategy (A), to give the same answer no matter what question you are asked
  - (3) and (4) are both versions of strategy (B), to give the same answer to 30? and 60?, but a different answer to 0?
  - ...

How Often Should You Choose Each Strategy?

- So we have four strategies, (A)–(D)
- In order to ensure that you and your partner always give the same answer if you are asked the same question, both of you must pick the same strategy as each other in each round
- The only thing left up to you is how often you choose each strategy
- Let's use α, β, γ and δ for the proportion of times you choose (A), (B), (C) and (D)
- Clearly, α, β, γ and δ must all be positive (or 0), and $\alpha + \beta + \gamma + \delta = 1$

How Often Should You Choose Each Strategy?

(1)	$\langle P, P, P \rangle$	(2)	$\langle A, A, A \rangle$	(A)
(3)	$\langle A, P, P \rangle$	(4)	$\langle P, A, A \rangle$	(B)
(5)	$\langle P, A, P \rangle$	(6)	$\langle A, P, A \rangle$	(C)
(7)	$\langle P, P, A \rangle$	(8)	$\langle A, A, P \rangle$	(D)

- You know that you are meant to agree 0.25 of the time when one of you are asked 0? and the other is asked 60?
- In other words, you should disagree on 0.75 of these times
- Only strategies (B) and (D) involve you disagreeing when you are asked these questions
- So you should choose (B) and (D) 0.75 of the time: $\beta + \delta = 0.75$

How Often Should You Choose Each Strategy?

(1)	$\langle P, P, P\rangle$	(2)	$\langle A, A, A\rangle$	(A)
(3)	$\langle A, P, P\rangle$	(4)	$\langle P, A, A\rangle$	(B)
(5)	$\langle P, A, P\rangle$	(6)	$\langle A, P, A\rangle$	(C)
(7)	$\langle P, P, A\rangle$	(8)	$\langle A, A, P\rangle$	(D)

- You know that you are meant to agree 0.75 of the time when one of you are asked 0? and the other is asked 30?
- In other words, you should disagree on 0.25 of these times
- Only strategies (B) and (C) involve you disagreeing when you are asked these questions
- So you should choose (B) and (C) 0.25 of the time: $\beta + \gamma = 0.25$

How Often Should You Choose Each Strategy?

(1)	$\langle P, P, P \rangle$	(2)	$\langle A, A, A \rangle$	(A)
(3)	$\langle A, P, P \rangle$	(4)	$\langle P, A, A \rangle$	(B)
(5)	$\langle P, A, P \rangle$	(6)	$\langle A, P, A \rangle$	(C)
(7)	$\langle P, P, A \rangle$	(8)	$\langle A, A, P \rangle$	(D)

- You know that you are meant to agree 0.75 of the time when one of you are asked 30? and the other is asked 60?
- In other words, you should disagree on 0.25 of these times
- Only strategies (C) and (D) involve you disagreeing when you are asked these questions
- So you should choose (C) and (D) 0.25 of the time: $\gamma + \delta = 0.25$

But That's Impossible!

(1) $\beta + \delta = 0.75$

(2) $\beta + \gamma = 0.25$

(3) $\gamma + \delta = 0.25$

$$(\beta + \gamma) + (\gamma + \delta) = 0.25 + 0.25 = 0.5$$

$$(\beta + \gamma) + (\gamma + \delta) = 2\gamma + (\beta + \delta) = 2\gamma + 0.75$$

$$0.5 = 2\gamma + 0.75$$

$$2\gamma = -0.25$$

$$\gamma = -0.125$$

But γ can't be a negative number!!!

Bell's Result

- What this shows is that if you and your partner cannot communicate with each other when you are being asked your questions, then you cannot replicate the behaviour of the photons, as predicted by QM
- But if Locality is true, then the photons cannot 'communicate' with each other
  - The events are space-like separated, and according to Locality, no causal influence can propagate faster than the speed of light
- **Bell's Result**: If Locality is true, then photons cannot behave in the way that QM predicts

  Disclaimer: this is a modified, and highly simplified, version of Bell's Result

The Significance of Bell's Result

- EPR made two assumptions their argument that QM was incomplete:
  (1) **QM is accurate:** photons behave as QM predicts
  (2) **Locality:** no causal influence can propagate faster than the speed of light

- But Bell's Result shows us that these two assumptions are inconsistent:
  - If Locality were true, then photons could not possibly behave in the way that QM predicts

- The EPR argument that QM is incomplete therefore collapses

A Possible Misunderstanding

- The significance of Bell's Result is sometimes misunderstood
- Because Einstein advocated a local **deterministic hidden-variables** theory, Bell framed his work as an exploration of the prospects for such a theory
- Thus it is often said that Bell showed that there could be no local deterministic hidden-variables theory
- That can make it sound like there is still a chance of giving some kind of local theory, just not a local theory which is **also** deterministic **and** involves hidden-variables
- **But that is a mistake: Bell showed that Locality by itself is inconsistent with the behaviour of photons, as predicted by QM!**

EPR and the Bell Results

Putting QM to the Test

- We have learnt from Bell that QM predicts results that are incompatible with Locality
- But we have not yet discussed whether QM's predictions are actually right
- In 1982, Aspect, Dalibard and Roger published a result showing that QM's predictions are correct: photons do behave in the way QM describes

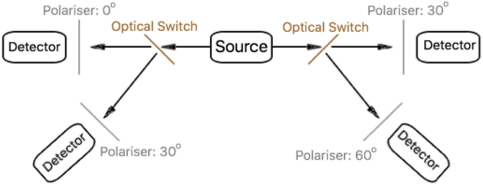
A Difficulty for Testing QM

- One problem with trying to set up such a test is that if you arranged two polarising filters too far in advance of performing the experiment, then it might be that our choice of how to set the filters somehow affected the behaviour of the photons

> *[It is conceivable that the QM predictions] might apply only to experiments in which the settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light.*
>
> *(Bell 1965 p. 407)*

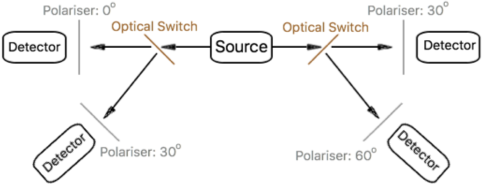
- Aspect's experiment was designed to rule out the possibility of this kind of 'collusion' between the measuring instruments and the particles

Setting Up Aspect's Experiment



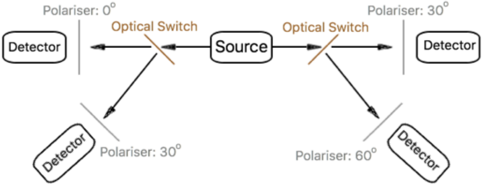
- A source emits two entangled photons
- On each side there is a switch which can change the path of the photons
- These switches can be flipped *very quickly* (once every 10^{-8} seconds)

Setting Up Aspect's Experiment



- One path on the left leads the photon to a polariser set at 0°, the other to a polariser at 30°
- One path on the right leads the photon to a polariser set at 30°, the other to a polariser at 60°

Setting Up Aspect's Experiment



- Behind all of these polarisers are photon detectors to check whether the photon passed through or was absorbed
- The detectors on the left are far enough away from the detectors on the right (approx 12m) to guarantee that there was a space-like separation between detection-events on each side

Apsect's Results

- **Aspect found that the predictions of QM were correct**
- If the degree of misalignment between filters is 0° and one photon passes, the probability that the other photon passes is 1
- If the degree of misalignment between filters is 30° and one photon passes, the probability that the other photon passes is 0.75
- If the degree of misalignment between filters is 60° and one photon passes, the probability that the other photon passes is 0.25

A Victory for Copenhagen

- We can conclude that any theory (whether deterministic or not, whether involving hidden variables or not) which satisfies Locality predicts results that are incompatible with the observed behaviour of photons
- If we liked, we could still insist that QM is incomplete, and that quantum particles have a full set of properties
  - Bohm presents such a theory, but is very clearly non-local
- However, we cannot use the EPR argument against Copenhagen!

The Completeness of QM and the Relativity of Simultaneity

- If we accept that QM is complete, then we have to grant that measuring the polarisation of one photon in a pair **changes** the other photon
- The other photon goes from not having a definite polarisation to having one
- But when does this change occur?

The Completeness of QM and the Relativity of Simultaneity

- We want to say: *Simultaneously with the measurement of the other photon!*
- But in SR, simultaneity is always relative to some frame of reference
- If we stick by this, then we will have to say that whether a photon has a polarisation can change from one frame to another
- It is an open question how best to reconcile QM and SR

References

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