

Paradoxes
Lecture Nine

Dialetheism

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Dialetheism

Introducing Dialetheism

What's So Bad About Contradictions?

The Logic of Paradox

Dialetheism and Three Paradoxes

Objections to Dialetheism

Two Stubborn Paradoxes

- **Russell's Paradox:**

(NC) For any condition F , the set $\{x : Fx\}$ exists

∴ $\{x : \sim x \in x\}$ (or R for short) exists

∴ $R \in R$ iff $\sim R \in R$

∴ $R \in R$ and $\sim R \in R$

- **The Liar Paradox:**

$\lambda = \text{'}\lambda \text{ is not true'}$

(T) ' p ' is true iff p

∴ ' λ is not true' is true iff λ is not true

∴ λ is true iff λ is not true

∴ λ is true and λ is not true

True Contradictions?

- Up until now, we have said that paradoxes like these **cannot** be veridical: contradictions cannot be true!
 - Although it is hard, we must uncover one of the premises as faulty, or point out a mistake in the reasoning
- But were we really right to insist that these paradoxes could not be veridical?
- Maybe we can just accept their contradictory conclusions, and carry on with our lives?

The Janus Headed Liar



Might one not even begin logic with this contradiction [the Liar]? And as it were descend from it to propositions.

The proposition that contradicts itself would stand like a monument (with a Janus head) over the propositions of logic.

(Wittgenstein, Remarks on the Foundations of Mathematics, IV. 59)

Dialetheism

- All contradictions are false, so a true contradiction would be both true **and** false
- Today, the philosopher most closely associated with the doctrine that some sentences are true and false is Graham Priest
- He calls this doctrine **dialetheism**, and he calls sentences which are both true and false **dialetheia**



Graham Priest

Dialetheism

The inspiration for the name was a passage in Wittgenstein's Remarks on the Foundations of Mathematics, where he describes the Liar sentence ('This sentence is not true') as a Janus-headed figure facing both truth and falsity (1978, IV. 59). Hence a di-aletheia is a two(-way) truth.

(Priest 2008, SEP entry on 'Dialetheism')

Dialetheism

- Dialetheism is a radical response to paradoxes like the Liar and Russell's Paradox
- It is so extreme that it might strike you as absurd
- But even if that is true, we need to look more closely at dialetheism, if only to put our finger on what is wrong with it
- As we will see, that is a surprisingly hard thing to do!

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Contradictions Entail Everything

- This is how we explain what entailment is in classical logic:
 - A entails B iff there is no interpretation which makes A true but does not make B true
- In classical logic, no interpretation makes any contradictions true
- It follows that in classical logic, contradictions entail **everything**

$A \& \sim A \therefore B$

- Consider any contradiction, $A \& \sim A$, and any other sentence you like, B
- In classical logic, no interpretation makes $A \& \sim A$ true
- So there is no interpretation which makes $A \& \sim A$ true **and** does not make B true
- So $A \& \sim A$ entails B
 - $A \& \sim A$ entails B iff there is no interpretation which makes $A \& \sim A$ true but does not make B true

Explosion

- The rule that contradictions entail everything was once known as *ex falso quodlibet*, which is Latin for: from falsehood, anything
- But that's a bit misleading, because it is only **logical** falsehoods (i.e. contradictions) which entail everything
- Nowadays, the rule is often called **Explosion**
 - Contradictions explode our theories, because they make them entail every single sentence in the language

Dialetheism versus Trivialism

- Logicians often use Explosion to explain what is wrong with contradictions
 - If a contradiction were true, then **every other** sentence would be true!
 - If you believe a contradiction, then you should believe everything!
- Priest calls the doctrine that *every* sentence is true **trivialism**
- Trivialism is (I take it!) obviously absurd
- The trouble for Priest is that if we accept Explosion, then we will have to say that dialetheism entails trivialism

Non-Classical Entailment

- Priest must somehow tweak our classical explanation of entailment, so that contradictions no longer entail everything
- Priest could try to do this by changing the classical definition of entailment
 - A entails B iff there is no interpretation which makes A true but does not make B true
- But Priest prefers to keep that definition, and change how we think of interpretations
 - On the classical conception, no interpretation can make a contradiction true
 - Maybe we can think of interpretations in a new way, so that interpretations **can** make contradictions true after all?

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Introducing the Logic of Paradox

- A logic which does not include the Explosion rule is called a **paraconsistent logic**
- It is worth noting that is not just dialetheists like Priest who are interested in paraconsistent logics
 - Many logicians think that classical logic is wrong because they think that A can entail B only if A is somehow **relevant** to B
 - Clearly, Explosion does not respect this idea, because **every** contradiction entails **every** sentence
- However, we are going to focus on Priest's own brand of paraconsistent logic, known as the **Logic of Paradox** (LP)
 - What I will present below is a simplified version of Priest's (1998) account

Classical Interpretations

- On the classical understanding, every sentence is either true or false, *but not both*
- A **classical interpretation** is an assignment of truth-values to every sentence
- We start off by assigning truth-values to the simplest sentences in the language (aka **atomic sentences**)
- We can assign them in any way we like, so long as every atomic sentence is made true or false but not both
- We then have some rules which tell us how to assign truth-values to complex sentences

Complex Sentences

$\sim A$ is true iff: A is false

$\sim A$ is false iff: A is true

$A \& B$ is true iff: A is true and B is true

$A \& B$ is false iff: A is false or B is false (or both)

$A \vee B$ is true iff: A is true or B is true (or both)

$A \vee B$ is false iff: A is false and B is false

$A \supset B$ is true iff: A is false or B is true (or both)

$A \supset B$ is false iff: A is true and B is false

$A \equiv B$ is true iff: A and B are both true, or they are both false

$A \equiv B$ is false iff: A is true and B is false, or A is false and B is true

Contradictions cannot be True

- It is easy to see that these rules will never allow a contradiction to be true
- For *reductio*, suppose that $A \& \sim A$ is true
 - By the rules for conjunction, that means A is true and $\sim A$ is true
 - By the rules for negation, if $\sim A$ is true, then A is false
 - So A is true and false
 - But classical interpretations **never** make one and the same sentence true and false!
- So $A \& \sim A$ can never be true on a classical interpretation, which is why contradictions entail everything in classical logic

LP Interpretations

- The interpretations for Priest's LP are just like classical interpretations, but with one important difference
- Interpretations are now allowed to make atomic sentences both true *and* false
- Everything else stays the same, including all the rules for deciding the truth-values of more complex sentences

Complex Sentences

$\sim A$ is true iff: A is false

$\sim A$ is false iff: A is true

$A \& B$ is true iff: A is true and B is true

$A \& B$ is false iff: A is false or B is false (or both)

$A \vee B$ is true iff: A is true or B is true (or both)

$A \vee B$ is false iff: A is false and B is false

$A \supset B$ is true iff: A is false or B is true (or both)

$A \supset B$ is false iff: A is true and B is false

$A \equiv B$ is true iff: A and B are both true, or they are both false

$A \equiv B$ is false iff: A is true and B is false, or A is false and B is true

Contradictions *can* be True

- Now that atomic sentences can be true *and* false, it is easy to see that these rules will not stop a contradiction being true
- If P is both true and false, then $P \& \sim P$ is true
 - Since P is false, the rule for negation tells us that $\sim P$ is true
 - So P is true and $\sim P$ is true, and the rules for conjunction tell us that this makes $P \& \sim P$ true too
- **Equally:** If P is both true and false, then $P \& \sim P$ is false
 - If P is false then the rules for conjunction make $P \& \sim P$ false too
- So in LP, if P is both true and false, then $P \& \sim P$ is both true and false too!

LP Entailment

- LP uses exactly the same definition of entailment as classical logic:
 - A entails B iff there is no interpretation which makes A true but does not make B true
- But now that we have changed how we are thinking about interpretations, this does not lead to Explosion
- Contradictions **do not** entail everything in LP!

Explosion Fails

- Let P and Q be any two atomic sentences you like
- There will be an interpretation, I , which behaves like this:
 - (i) P is both true and false
 - (ii) Q is false, but not true
- From (i) and the rules for negation and conjunction, we can infer $P \& \sim P$
- So $P \& \sim P$ is true on I , but Q is not true on I
- So $P \& \sim P$ does not entail Q
- So Explosion fails!

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Accepting Contradictions

- The logical machinery we have just been working through (LP) prevents dialetheism (some contradictions are true) from collapsing into trivialism (all sentences are true)
- But it does not yet show that dialetheism is right
- Why exactly should we accept that some contradictions are true?
- **Priest's Answer:** because doing so gives us particularly simple, elegant and satisfying responses to various paradoxes

The Liar Paradox

(T) ' p ' is true $\equiv p$

(F) ' p ' is false $\equiv \sim p$

- Most consistent solutions to the Liar Paradox involving somehow restricting (T) and (F)
- But if we are just happy to accept the conclusion of the Liar Paradox (λ is true and not true), then we can leave (T) and (F) as they are
- Priest suggests that this may be the most intellectually economical response

Virtues of Inconsistency

there are criteria for rationality other than consistency, and [...] some of these are even more powerful than consistency. The point is, in fact, a familiar one from the philosophy of science. There are many features of belief that are rational virtues, such as simplicity, problem-solving ability, non-adhocness, fruitfulness, and, let us grant, consistency. However, these criteria are all independent, and may even be orthogonal, pulling in opposite directions

Virtues of Inconsistency

Now what should one do if, for a certain belief, all the criteria pull towards acceptance, except consistency — which pulls the other way? It may be silly to be a democrat about this, and simply count the number of criteria on each side, but it seems natural to suppose that the combined force of the other criteria may trump inconsistency. In such a case, then, it is rational to have an inconsistent belief [...]

The Liar Paradox

The following is a simple account of truth. Truth is a principle that is characterized formally by the T-schema: for every sentence α , $T(\alpha) \leftrightarrow \alpha$ (for suitable conditional connective). And that's an end on't. (There may be more to be said about truth, but nothing that can be captured in a formalism.) This account is inconsistent: when suitable self-referential machinery is present, say in the form of arithmetic, the Liar paradox is forthcoming. Yet the inconsistencies are isolated. In particular, it can be shown that, when things are suitably set up, inconsistencies do not percolate into the purely arithmetic machinery. In fact, it can be shown that any sentence that is grounded (in Kripke's sense) behaves consistently [...]

The Liar Paradox

What are the alternatives to such an account? There is a welter of them: Tarski's, Kripke's, Gupta and Herzberger's, Barwise and Perry's, McGee's, etc., etc. These may all have the virtue of consistency, but the other virtues are thinly distributed amongst them. They often have strong ad hoc elements; they are complex, usually involving transfinite hierarchies; they have a tendency to pose just as many problems as they solve; and it is not clear that, in the last instance, they really solve the problem they are supposed to: they all seem subject to extended paradoxes of some kind. It seems to me that rationality speaks very strongly in favour of the simple inconsistent theory.

(Priest 1998 pp. 32–3)

Russell's Paradox

- Similarly, in response to Russell's paradox, we may accept the conclusion as another dialethia: $R \in R$ and $\sim R \in R$
- This has the virtue of letting us maintain Naïve Comprehension:
(NC) For any condition F , the set $\{x : Fx\}$ exists
- This is a particularly simple and elegant foundation for set theory, and a case could be made that it fits our intuitions better than anything consistent
 - For more, see Beall 2004, pp. 9–10

The Sorites Paradox

- Some people have even suggested that we could apply dialetheism to the Sorites Paradox
- We are now very familiar with the idea that we might respond to the Sorites paradox with truth-value gaps
 - When a is a borderline case of F , ' a is F ' is neither true nor false
- We might alternatively respond by positing dialethia
 - When a is a borderline case of F , ' a is F ' is both true and false
- Beall (2004 p.11) says: "In the end, it seems initially reasonable to think that a 'vague language' is *overdetermined* as to think it is *underdetermined*"

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Some Common Objections

- We've already dealt with two objections to dialetheism
 - (1) Contradictions entail everything!
 - Not in a paraconsistent logic, like LP
 - (2) It is never rational to believe a contradiction!
 - Not if Priest is right, and intellectual virtues other than consistency can make it worth accepting inconsistent solutions to various paradoxes
- I will now look at two more objections:
 - (3) Contradictions just cannot be true!
 - (4) If contradictions were acceptable, people could never be rationally criticised!

Contradictions Just Cannot Be True!

- This is really more of an angry rejection of dialetheism than an argument against it
- Still, there is a lot of intuitive force to this complaint, and dialetheists need to say something about it
- One thing Priest does (1998) is look at various arguments for the idea that contradictions cannot be true, and argues that none of them work
- But he also has a more interesting dialethianist response...

The Law of Non-Contradiction

- **The Law of Non-Contradiction:** $\sim (A \& \sim A)$
- LNC is a classical law, but what is its status in LP
- You might have thought that it couldn't possibly be a law of LP, but if so, you would be wrong!
 - Either A is a dialethia, or it is not
 - If A is not a dialethia, then $A \& \sim A$ is false, not true, and so $\sim (A \& \sim A)$ is true, not false
 - If A is a dialethia, then $A \& \sim A$ is true, but it is *also* false
 - So if A is a dialethia, then $\sim (A \& \sim A)$ is true (as well as false)

Contradictions Just Cannot Be True!

- Priest accepts $\sim (A \& \sim A)$, no matter what sentence A is
- We also saw that one of the main selling points of dialetheism is that it let's us stick to a completely unrestricted version of (T):
(T) ' p ' is true $\equiv p$
- So, Priest accepts this:
 - ' $A \& \sim A$ ' is true iff $A \& \sim A$
- When we combine this with $\sim (A \& \sim A)$, we can infer:
 - ' $A \& \sim A$ ' is not true
- In other words, contradictions are never true!

Sentences Just Cannot Be Both True And False!

- You can give a similar argument to show that no sentence can be true and false
- This time we need to use the truth schema **and** the falsity schema:
 - (T) ' p ' is true $\equiv p$
 - (F) ' p ' is false $\equiv \sim p$
- These schemata together turn $\sim(A \ \& \ \sim A)$ into:
 - $\sim ('A' \text{ is true} \ \& \ 'A' \text{ is false})$

Dialetheism about Dialetheism

- So, using principles that Priest accepts, we can infer that his dialetheism is false:
 - No contradiction is true
 - No sentence is both true and false
- Does this spell the end for dialetheism?
- No! The dialetheist just thinks that dialetheism is **true and false!**
- As Priest (2008) puts it: “a dialetheist manifests her dialetheism in accepting, together with LNC, sentences that are inconsistent with it”

Rational Criticism

- At this point you might start to worry: if we can accept contradictions are true, and that some sentences are true and false, how could we ever argue with anyone?
- Normally we criticise someone for believing A by arguing for the truth of $\sim A$
- But now we can just accept both A and $\sim A$ together!

Rational Criticism

- Priest tries to downplay the importance of this worry by pointing out that the dialetheist does not want to accept **every** contradiction, so it is not **always** acceptable to respond to criticism by accepting a contradiction
- But the trouble is, we have no clear account of **when** it is OK to accept a contradiction
- So even if we all agree that we will not **always** use dialetheism to dodge a criticism, it is always open to someone to respond to a criticism by using dialetheism **this time**

The Resilience of Dialetheism

*It is difficult for the critic to know what to say. One can refute other opponents by showing that their views lead to contradiction, especially if the inconsistency comes by the opponent's own lights. The debate usually turns on whether the view does in fact entail a contradiction. This cuts no ice against the dialetheist, since she embraces contradictions. For her, a **reductio ad contradictionem** is not a **reductio ad absurdum**.*

(Shapiro 2004 p. 337)

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