Paradoxes
Lecture Five

Vagueness: The Sorites Paradox

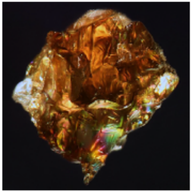
Rob Trueman
rob.trueman@york.ac.uk

University of York

Vagueness: The Sorites Paradox

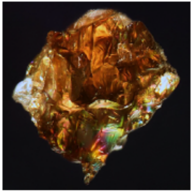
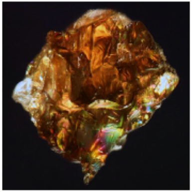
The Paradox of the Heap

- 1 grain of sand does not make a heap



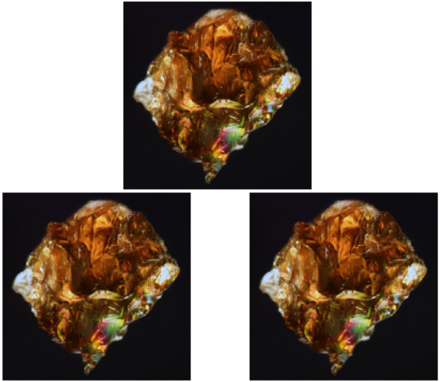
The Paradox of the Heap

- If 1 grain of sand does not make a heap, then 2 grains of sand do not make a heap

The Paradox of the Heap

- If 2 grains of sand do not make a heap, then 3 grains of sand do not make a heap



The Paradox of the Heap

999,999,997 steps later...

The Paradox of the Heap

- If 999,999,999 grains of sand do not make a heap, then 1,000,000,000 grains of sand do not make a heap

The Paradox of the Heap

- So 1,000,000,000 grains of sand do not make a heap!

Vagueness

- The Paradox of the Heap exploits the fact that *heap* is a vague concept
- We can set up a Sorites paradox for any vague concept
  - Bald
  - Tall
  - Table (!?)

General Form A

- $\phi(0)$
- $\forall n[\phi(n) \supset \phi(n+1)]$ (QP)
- $\therefore \forall n \phi(n)$

General Form B

- $\phi(0)$
- $\phi(0) \supset \phi(1)$
- $\phi(1) \supset \phi(2)$
- ...
- $\phi(m-1) \supset \phi(m)$
- $\therefore \phi(m)$

- m is a sufficiently large number that it is obviously false to say $\phi(m)$

Vague Concepts as Faulty Concepts

- One way of responding to the Sorites paradoxes is to say that vague concepts are inherently flawed, and so cannot be used to describe the world
- This was Unger's view: there are no heaps or tables, and nobody is bald or tall
- Frege held a similar view, but went even further: vague concepts are so faulty that they cannot even be meaningfully applied at all!

Epistemicism

- Another way of responding to the paradox is by denying one of its conditional premises:

$$\sim[\phi(n) \supset \phi(n+1)]$$
$$\phi(n) \; \& \sim\phi(n+1)$$

- In other words, it involves asserting that there is a sharp cut off line between ϕ and $\sim\phi$
- This might sound absurd: what is the cut off between being bald and being not-bald?
- According to Williamson's **epistemicism**, there is an answer to that question; it's just that we **cannot know** the answer

Comparing these two Responses

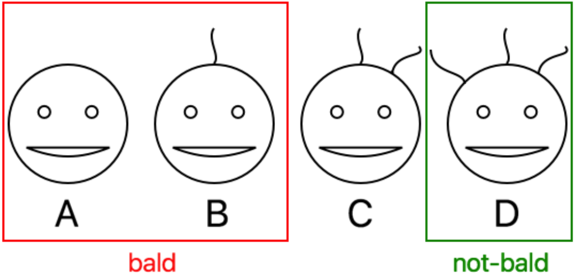
- Unger's "vague concepts are faulty concepts" view is in many ways similar to Williamson's epistemicism
- They both deal with the Sorites paradoxes without complicating our semantic theories or our logical rules
- But in a certain sense, they both refuse to take vagueness seriously
  - Unger: vague concepts are just broken concepts
  - Williamson: to say that a concept is vague is just to say that we do not know where its boundary lies
- This week, we will look at **supervaluationism**, a response to the Sorites paradox which takes vagueness more seriously

Vagueness: The Sorites Paradox

Vagueness as Semantic Imprecision

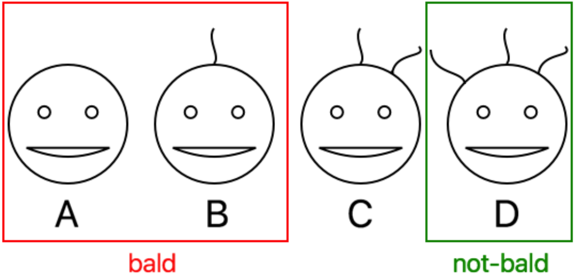
- The way we use the word ‘bald’ makes it clearly right to call some people bald
- The way we use the word ‘bald’ makes it clearly right to call some people not-bald
- But there are **borderline cases**, people who are not rightly called bald and who are not rightly called not-bald
- This is because the way we use the word ‘bald’ just isn’t precise enough to settle in each and every case whether someone is bald or whether they are not-bald

Vagueness as Semantic Imprecision



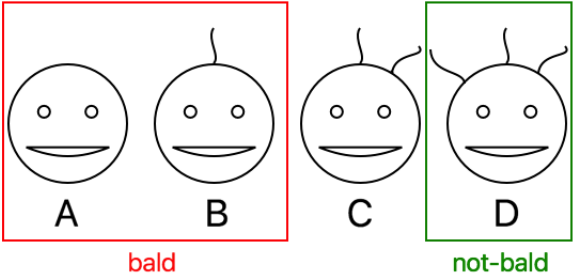
- The way we use the word 'bald' makes it right to call A and B bald

Vagueness as Semantic Imprecision



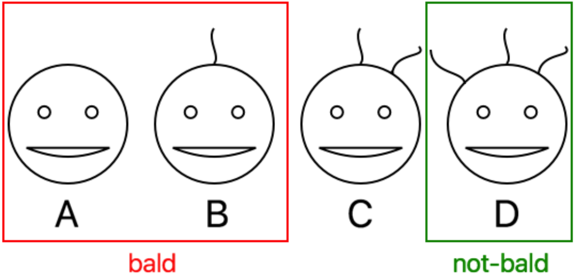
- And the way we use the word 'bald' makes it right to call D not-bald

Vagueness as Semantic Imprecision



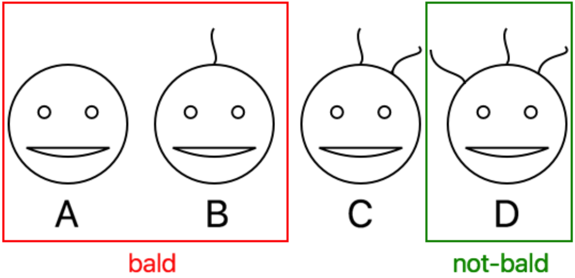
- But the way we use the word ‘bald’ does not make it right to call C bald or to call C not-bald

Vagueness as Semantic Imprecision



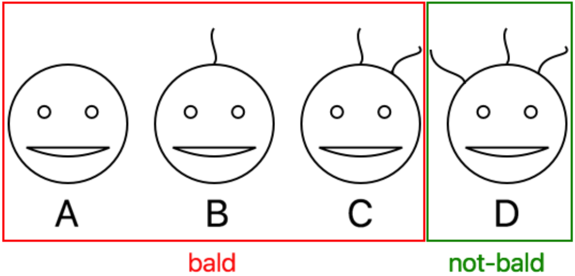
- The way we use the word ‘bald’ is too imprecise to do that

Making Vague Concepts more Precise



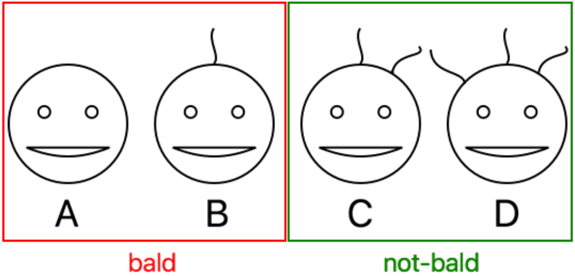
- But we could make our use of the word ‘bald’ more precise, if we liked

Making Vague Concepts more Precise



- We could make stipulate that from now on, C will count as bald

Making Vague Concepts more Precise



- Or we could make stipulate that from now on, C will count as not-bald

Precisifications

- These ways of making ‘bald’ more precise are known as **precisifications** of ‘bald’
- These precisifications assign a precise extension to ‘bald’
  - The extension of a predicate is the set of things which that predicate applies to
  - So when we give a precisification of ‘bald’, we are picking a determinate set of things, and stipulating that ‘bald’ is to be true of just those things

Precisifications

- Every admissible precisification of ‘bald’ needs to conform to the following rules:
  - If x is clearly bald, then x must be in the extension of ‘bald’
  - If x is clearly not-bald, then x must not be in the extension of ‘bald’
  - If x is in the extension of ‘bald’ and y is more bald (i.e. has fewer hairs) than x, then y must be in the extension of ‘bald’ too
  - If x is not in the extension of ‘bald’ and x is more bald than y, then y must not be in the extension of ‘bald’ either
- There will be lots of different precisifications of ‘bald’ which conform to all of these rules

Supervaluationism

- According to **supervaluationism**, when we want to figure out whether a claim involving the word 'bald' is true or false, we need to look at *all* the different precisifications of 'bald'
- 'Paul is bald' is true if and only if it is true on all admissible precisifications of 'bald'
- 'Paul is bald' is false if and only if it is false on all admissible precisifications of 'bald'
  - 'P' is false iff '$\sim P$' is true

Supervaluationism

- And of course, what goes for the predicate ‘bald’ goes for **all** vague predicates

- A precisification of a whole language, *L*, assigns precise extensions to **all** the predicates in *L*, obeying the rules on admissible precisifications given earlier:

  - If *x* is clearly *F*, then *x* must be in the extension of ‘*F*’
  - If *x* is clearly not-*F*, then *x* must not be in the extension of ‘*F*’
  - If *x* is in the extension of ‘*F*’ and *y* is more *F* than *x*, then *y* must be in the extension of ‘*F*’ too
  - If *x* is not in the extension of ‘*F*’ and *x* is more *F* than *y*, then *y* must not be in the extension of ‘*F*’ either

Supervaluationism

- A is true if and only if A is true on every admissible precisification of the language

- A is false if and only if A is false on every admissible precisification of the language

- Supervaluationism has been developed and defended by many philosophers, but see especially Fine's (1975) and Keefe's (2000)

Bivalence Lost

- Supervaluationism gives up on **bivalence**, according to which every sentence is either true or false
- Suppose that Paul is neither clearly bald, nor clearly not-bald
- 'Paul is bald' is not true
  - Since Paul is not clearly bald, there will be some precisification of 'bald' on which 'Paul is bald' is not true
- 'Paul is bald' is not false
  - Since Paul is not clearly not-bald, there will be some precisification of 'bald' on which 'Paul is bald' is true, and hence not false

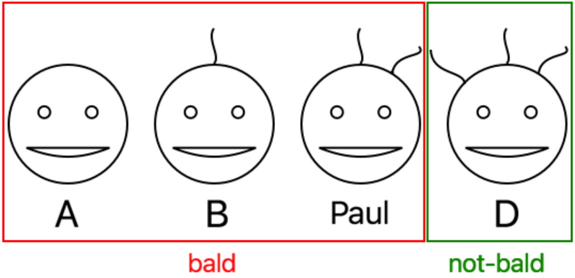
LEM Retained

- But interestingly, supervaluationism does not need to give up on the **Law of Excluded Middle** (LEM):

$$P \vee \sim P$$

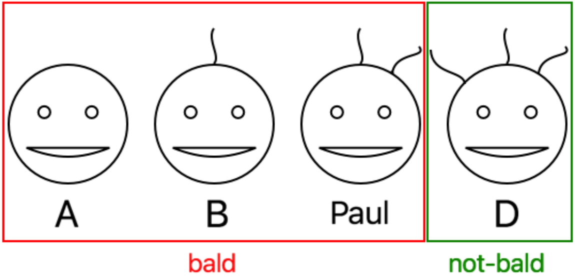
- ‘Paul is bald or not-(Paul is bald)’ is true on every precisification!

LEM Retained



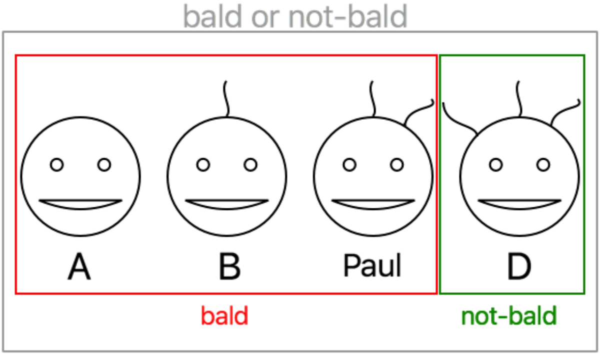
- Consider the precisifications which do put Paul in the extension of ‘bald’

Bivalence versus LEM



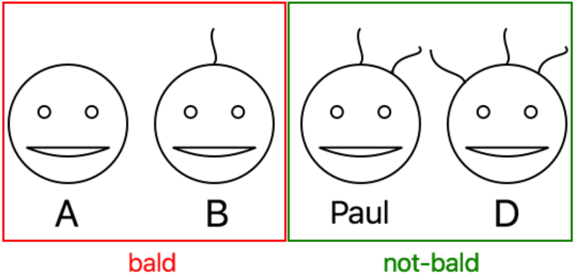
- ‘Paul is bald’ is true on these precisifications...

Bivalence versus LEM



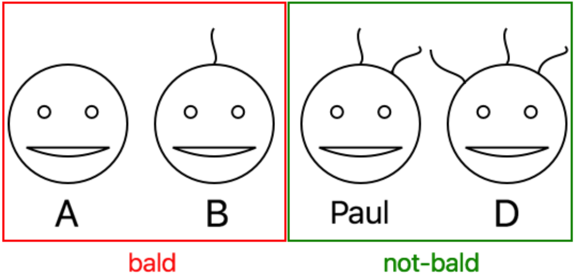
- ...and thus so is 'Paul is bald or not-(Paul is bald)'

Bivalence versus LEM



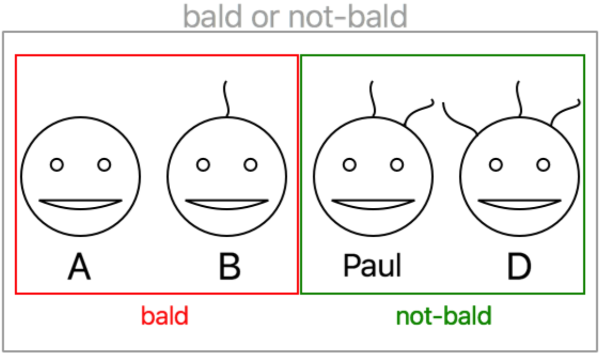
- Now consider the precisifications which do **not** put Paul in the extension of 'bald'

Bivalence versus LEM



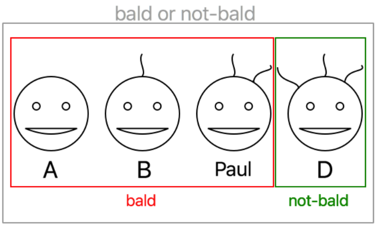
- ‘not-(Paul is bald)’ is true on these precisifications...

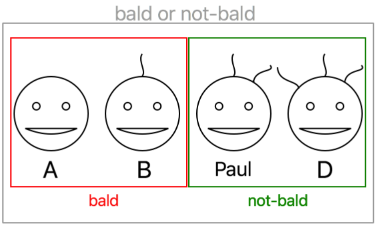
Bivalence versus LEM



- ...and thus so is ‘Paul is bald or not-(Paul is bald)’

Bivalence versus LEM





- Therefore ‘Paul is bald or not-(Paul is bald)’ is true on all the admissible precisifications

Bivalence versus LEM

- Thus, LEM is *not* undermined by supervaluationism
- This is good, since LEM is a classical law of logic
- And in fact, supervaluationism is compatible with **all** the classical laws of logic

Vagueness: The Sorites Paradox

Form B

- Let's start with Form B of the Sorites paradoxes:

  A man with 0 hairs is bald

  If a man with 0 hairs is bald, then a man with 1 hair is bald

  If a man with 1 hair is bald, then a man with 2 hairs is bald

  ...

  If a man with 9,999 hairs is bald, then a man with 10,000 hairs is bald

  Therefore a man with 10,000 hairs is bald

- According to supervaluationism, at least one of the conditional premises will not be true

Form B

- Suppose that people with 23 hairs and people with 24 hairs are **borderline cases** of being bald:
  - They are neither clearly bald, nor clearly not-bald
- 'If a man with 23 hairs is bald, then a man with 24 hairs is bald' is not true, because it is not true on every admissible precisification
  - Since 23 and 24 are both borderline cases, there will be an admissible precisification of 'bald' which puts people with 23 hairs in the extension of 'bald', but not people with 24 hairs
  - On this precisification, 'If a man with 23 hairs is bald, then a man with 24 hairs is bald' is not true
- So for a supervaluationist, the Sorites paradoxes are premise-flawed, in the sense that they rely on untrue premises

Sharp Boundaries Again?

- Earlier I said that in classical logic, denying this conditional:
  - If a man with 23 hairs is bald, then a man with 24 hairs is bald

  is equivalent to asserting this conjunction:
  - A man with 23 hairs is bald, and a man with 24 hairs is not bald
- I also said that supervaluationism obeys classical logic
- So doesn't supervaluationism have to say that there is a sharp cut-off line between being bald and not-bald?
- No!

Neither True nor False!

- When I said that denying '$P \supset Q$' is equivalent to asserting '$P \& \sim Q$', I meant that calling '$P \supset Q$' **false** is equivalent to calling '$P \& \sim Q$' **true**
- But according to supervaluationism, this conditional is not true *or false*:
  - If a man with 23 hairs is bald, then a man with 24 hairs is bald
- We have already seen why it is not true, and it is easy to see that it also is not false:
  - There will be some admissible precisification of 'bald' which puts people with 23 hairs and people with 24 hairs in the extension of 'bald'
  - On these precisifications, 'If a man with 23 hairs is bald, then a man with 24 hairs is bald' is true, not false

Form A

- Now let's look at Form A of the Sorites paradoxes:

  A man with 0 hairs is bald

  (QP) $\forall n$ [a man with n hairs is bald $\supset$ a man with $n+1$ hairs is bald]

  Therefore $\forall n$[a man with n hairs is bald]

- According to supervaluationism, QP is not true

Form A

- If 23 hairs and 24 hairs are borderline cases of ‘bald’, then there will be admissible precisifications of ‘bald’ on which the following is *not true*:
  - A man with 23 hairs is bald $\supset$ a man with 24 hairs is bald
- But this conditional is an instance of QP for ‘bald’:
  - $\forall n$(a man with n hairs is bald $\supset$ a man with $n+1$ hairs is bald)
- So QP cannot be true either

QP is False!

- QP is not just not-true: it is false!
- On every precisification, there is a sharp line between people who are bald and people who are not
- Different precisifications draw this line in different places, but they all draw it somewhere
- So QP will turn out to be false on every single precisification!

Sharp Boundaries *Again!?*

- But in classical logic, saying that QP is false is equivalent to saying that this is true:
  - $\exists n$(a man with n hairs is bald and a man with $n+1$ hairs is not bald)

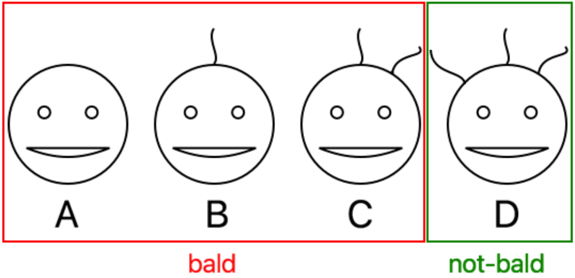
- This surely says that there is a sharp cut off between being bald and not somewhere, it just doesn't tell us where

- So now haven't the supervaluationists ended up positing sharp boundaries after all?

Existential Generalisations in Supervaluationism

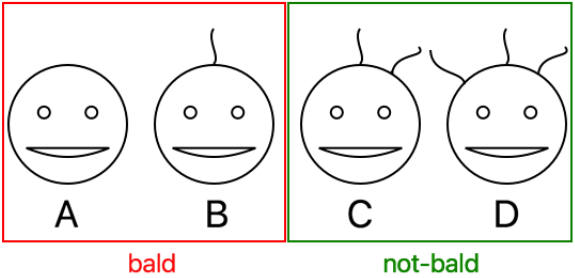
- We already saw that according to supervaluationism, **disjunctions** can be true even when both its disjuncts are not true:
  - 'Paul is bald or not-(Paul is bald)' is true
  - 'Paul is bald' is not true
  - 'not-(Paul is bald)' is not true
- In exactly the same way, according to supervaluationism, **existential generalisations** can be true even when the generalisation is not true of any particular thing
  - '$\exists n$(a man with n hairs is bald and a man with $n+1$ hairs is not bald)' is true
  - 'A man with n hairs is bald and a man with $n+1$ hairs is not bald' is not true of any number n

Existential Generalisations in Supervaluationism



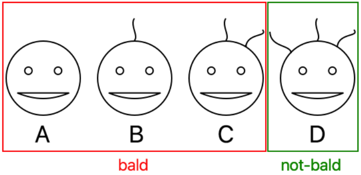
- On this precisification, there is some n such that a man with n hairs is bald, but a man with $n+1$ is not ($n=2$)

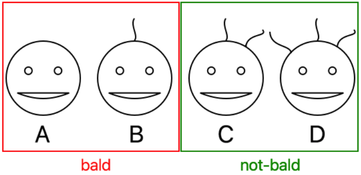
Existential Generalisations in Supervaluationism



- On this precisification, there is some n such that a man with n hairs is bald, but a man with $n+1$ is not ($n = 1$)

Existential Generalisations in Supervaluationism





- '$\exists n$(a man with n hairs is bald and a man with $n+1$ hairs is not bald)' is true on every precisification
- But since the number n changes from one precisification to another, there is not a single number which 'A man with n hairs is bald and a man with $n+1$ hairs is not' is true of on every precisification

Vagueness: The Sorites Paradox

Supervaluationism in Reivew

- Supervaluationism is very attractive
  - It takes vagueness very seriously, as a real property of various predicates
  - It preserves classical logic, in the sense of preserving classically valid inferences and obeying classical laws of logic
- However, it does have some weird features
  - Disjunctions can be true even when neither of their disjuncts are
  - Existential generalisations can be true even when the generalisation is not true of anything
- Perhaps we can live with these oddities, but there is another objection

Higher-Order Vagueness

- Admissible precisifications of ‘bald’ must put everyone who is clearly bald into the extension of ‘bald’
- But who is clearly bald?
- There is no sharp cut off line between being clearly bald and being not clearly bald
- The predicate ‘clearly bald’ is just as vague as the predicate ‘bald’
- In fact, we could set up a Sorites paradox for clearly bald:
  - A man with 0 hairs is clearly bald; if a man with n hairs is clearly bald, then so is a man with $n+1$ hairs; so a man with 10,000 hairs is clearly bald

Vague Semantics for Vague Languages

- It is not 100% clear how serious this problem is for supervaluationism
- It is really bad news if you wanted to give a non-vague semantics for a vague language
- But maybe we can accept that the semantics of a vague language has to be vague too?
- If they took this line, supervaluationists would just say that it is sometimes vague whether a precisification counts as admissible
  - For example: if Paul is a borderline case of being clearly bald, then a precisification which does not put him in the extension of ‘bald’ is a borderline case of being admissible

Vagueness: The Sorites Paradox

Not Taking Vagueness Seriously

- We have looked at three responses to the Sorites paradoxes
- Two of them didn't take vagueness very seriously
  - Unger's view that vague concepts are faulty concepts
  - Williamson's epistemicism
- The benefit of these views is that they do not call for any modifications to logic or semantics
- The downside of these views is that they just seem somewhat incredible
  - Are there really no tables?
  - Is there really an unknowable sharp boundary between being bald and being not-bald?

Taking Vagueness Seriously

- We also looked at a response which does take vagueness seriously: supervaluationism
- Supervaluationism is in many ways a very elegant framework for dealing with vagueness
- But it is open to a number of objections
  - Supervaluationism "preserves classical logic", but does so in a strange way: it allows disjunctions to be true even when they have no true disjuncts, and existential generalisations to be true even when they have no true instances
  - Supervaluationism faces a problem of higher-order vagueness

What are we to do?

- I think it's fair to say that it is not easy to pick between these options
- None of them are perfect!
- Good news (or perhaps bad): there are a whole lot more options to consider!
  - See the SEP article on the Sorites for some of them
  - I have also posted some bonus slides to the VLE on an alternative response: **Fuzzy Logic**

This Week's Seminar

- Please read the following articles for this week's seminar:
  - Williamson, T (1992) 'Vagueness and Ignorance', *Proceedings of the Aristotelian Society*
  - Keefe, R (2008) 'Vagueness: Supervaluationism', *Philosophy Compass*
- Both of these articles are available via the Reading List on the VLE
- I have also posted some study questions to the VLE. Please bring short **written** answers to those questions

Next Week

- We will be looking at Newcomb's Paradox
- Required Reading:
  - Sainsbury's *Paradoxes*, Chapter 4

References

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- Hyde, D 'Sorites Paradox', *The Stanford Encyclopedia of Philosophy* (Winter 2011 edition), Zalta, E (ed), URL: <http://plato.stanford.edu/archives/win2011/entries/sorites-paradox/>
- Keefe, R (2000) *Theories of Vagueness* (Cambridge: CUP)
- Sainsbury, RM and Williamson, T (1997) 'Sorites' in Hale and Wright (eds) *A Companion to the Philosophy of Language* (Oxford: Blackwell)
- Unger, P (1979) 'There are no ordinary things', *Synthese* 41: 117–54