

Paradoxes

Lecture Two

The Unexpected Exam

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The Unexpected Exam

The Unexpected Exam

A Veridical Paradox?

A Fallacious Paradox?

An Unepistemological Paradox

Exam Next Week!

- There will be an exam at 12 noon one day next week (Monday–Friday)
- **The exam will be a surprise!**
- You will not be able to figure out when the exam is going to happen until a few minutes before it is happening

What's on?

Mon	<input type="radio"/>
Tues	<input type="radio"/>
Wed	<input type="radio"/>
Thurs	<input type="radio"/>
Fri	<input type="radio"/>

What's on?

Mon	<input type="radio"/>
Tues	<input type="radio"/>
Wed	<input type="radio"/>
Thurs	<input type="radio"/>
Fri	<input type="radio"/> Test?

The Test cannot be on Friday

- The test can't be on Friday
- If it got to Thursday night and you still hadn't had the test, you could figure out that it would have to be on Friday!

What's on?

Mon	<input type="radio"/>	
Tues	<input type="radio"/>	
Wed	<input type="radio"/>	
Thurs	<input type="radio"/>	
Fri	<input type="radio"/>	X

What's on?

Mon	<input type="radio"/>	
Tues	<input type="radio"/>	
Wed	<input type="radio"/>	
Thurs	<input type="radio"/>	Test?
Fri	<input type="radio"/>	X

The Test cannot be on Thursday

- The test can't be on Thursday
- If it got to Wednesday night and you still hadn't had the test, you could figure out that it would have to be on Thursday or Friday
- But you have already figured out that it can't be on Friday
- So you could figure out that it would have to be on Thursday!

Paradoxes (2): The Unexpected Exam

└ The Unexpected Exam

What's on?

Mon	<input type="radio"/>	
Tues	<input type="radio"/>	
Wed	<input type="radio"/>	
Thurs	<input type="radio"/>	X
Fri	<input type="radio"/>	X

Paradoxes (2): The Unexpected Exam

└ The Unexpected Exam

What's on?

Mon	<input type="radio"/>	
Tues	<input type="radio"/>	
Wed	<input type="radio"/>	X
Thurs	<input type="radio"/>	X
Fri	<input type="radio"/>	X

Paradoxes (2): The Unexpected Exam

└ The Unexpected Exam

What's on?

Mon	<input type="radio"/>	
Tues	<input type="radio"/>	X
Wed	<input type="radio"/>	X
Thurs	<input type="radio"/>	X
Fri	<input type="radio"/>	X

What's on?

Mon	<input type="radio"/>	X
Tues	<input type="radio"/>	X
Wed	<input type="radio"/>	X
Thurs	<input type="radio"/>	X
Fri	<input type="radio"/>	X

Names of the Paradox

- This is a well-known paradox, and it has gone by a number of names:
 - The Unexpected Exam Paradox
 - The Surprise Test Paradox
 - The Class-A Blackout Paradox
 - The Hangman Paradox
 - The Prediction Paradox
 - The Surprise Egg Paradox

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An Initial Formulation of the Paradox

- (P₁) There will be a class test one day next week, but the students won't know what day the test is happening until a few minutes before it happens
- (C) There can be no such test
- This formulation can't be right
 - Imagine that the Teacher had decided to set this surprise exam, but kept the information to herself
 - In that case, the paradox wouldn't get going
 - We need to include the information that the students have been **told** that about the surprise exam!

An Initial Formulation of the Paradox

- (P₂) The Teacher has promised her students that there will be a class test one day next week, and that the students won't know what day the test is happening until a few minutes before it happens
- (C) There can be no such test
- The exam can't be on Friday
 - If the students got to Thursday night and hadn't been examined, they would know that the exam was going to be on Friday, but then it wouldn't be a surprise
 - By similar reasoning, the exam also can't be on Thursday, or Wednesday, or Tuesday or Monday

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Quinean Classifications

A paradox is an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises.

(Sainsbury's definition from Paradoxes, p. 1)

- Premise-flawed
 - One of the premises turns out to be false
- Fallacious (“Falsidical”)
 - The reasoning turns out to be faulty
- Veridical
 - The conclusion turns out to be true

Not Premise-Flawed!

- Premise-flawed
- Fallacious
- Veridical

Not Premise-Flawed!

- ~~Premise-flawed~~
- Fallacious
- Veridical

- There was **only one** premise in the argument:
 - (P₂) The Teacher has promised her students that there will be a class test one day next week, but the students won't know what day the test is happening until a few minutes before it happens

- Clearly, there is no problem with P₂ being true: it is the sort of thing teachers do all the time

A Veridical Paradox?

- The easiest thing to do here would just be to say that the paradox is **veridical**
- Since The Teacher announced her intentions to give her students a surprise exam, she cannot give her exam
- This is not to say that it is impossible for teachers to give surprise exams!
 - It is just that they cannot give them **and** announce them in the way that The Teacher did
- Maybe it would be surprising if The Teacher could not give a surprise exam after announcing her intention to, but maybe it wouldn't be a **very** surprising surprise

The Barber Paradox

- The Barber shaves **only** the people in his village who do not shave themselves
- The Barber shaves **all** the people in his village who do not shave themselves
- Does The Barber shave himself?



The Barber Paradox



- If The Barber shaves himself, then he **does not** shave himself
- If The Barber does not shave himself, then he **does** shave himself
- So The Barber both does and does not shave himself!

The Barber and The Teacher

- The Barber Paradox is a veridical paradox
 - It just shows us that no barber can shave all and only the people who do not shave themselves
- Of course, it does **not** show that no barber could try (and of necessity **fail**) to live by the policy of shaving all and only those who do not shave themselves
- It is tempting to say that the Unexpected Exam Paradox is a veridical paradox
 - It just shows us that no teacher can surprise their students with an exam after giving the kind of warning that The Teacher gave
- Of course, it does **not** show that no teacher could **try** to surprise their students after warning them

But Hold On!

- Let's suppose that The Teacher does give the students an exam on Tuesday
- The students would be very surprised indeed!
- They thought that they had **proved** that they couldn't be surprised by the exam

Logic Refuted?

It appears that the logical gadget which established this conclusion [that The Teacher can't keep her promise] has somehow short-circuited. I think this flavour of logic refuted by the world makes the paradox rather fascinating. The logician goes pathetically through the motions that have always worked the spell before, but somehow the monster, Reality, has missed the point and advances still.

Scriven (1951) p. 403

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Quine's Rejection of the Antinomy

What is remarkable is that the solution, a solution which at any rate has contented me for nine years, seems seldom to have been clearly apprehended. There is a false notion abroad that an actual antinomy is involved.

Quine (1953) p. 19

(An **antinomy** is a paradox we do not know how to resolve)

The Fallacy (according to Quine)

It is notable that the students acquiesce in the conclusion (wrong according to the fable of the Tuesday examination) that the promise cannot be fulfilled. If this is a conclusion which they are prepared to accept (though wrongly) at the end as a certainty, it is an alternative which they should have been prepared to take into consideration from the beginning as a possibility.

Quine (1953) p. 20 — modified to deal with the exam scenario

Spelling Quine's Point Out

- The students start from the assumption that The Teacher's promise will be kept
 - The examination will happen on Monday, Tuesday, Wednesday, Thursday or Friday
- They conclude at the end of their reasoning that the exam won't happen at all, **and so the promise won't be kept after all!**
- Quine's point is that they should factor the possibility of the promise's not being kept into their reasoning **right from the start**

How this Solves the Paradox

- On Thursday, the students should say to themselves:
 - **Either** The Teacher won't keep her promise, **or** there will be an exam on Friday
- Since the students don't **know** whether the teacher will keep her promise, they also don't know whether there will be an exam on Friday
- So if there is an exam on Friday, it will still be a surprise!

Objecting to Quine's Solution

- (P₁) There will be a class test one day next week, but the students won't know what day the test is happening until a few minutes before it happens
- (P₂) The Teacher has promised her students that there will be a class test one day next week, but the students won't know what day the test is happening until a few minutes before it happens
- At the start of the lecture, I emphasised that we need P₂ to get the Unexpected Exam Paradox going
 - However, Quine goes further and assumes that the **students** should start **their** reasoning from P₂, and that might seem mistaken

An analogy

- Imagine that Sharon saw that grass is green and so inferred that either grass is green or grass is blue
- Now consider the following two sentences:
 - (Q₁) Grass is green
 - (Q₂) Sharon looked at some grass, and it looked green
- When telling the story of how Sharon realised that grass is green or grass is blue, I have to say Q₂
- But it would be **absolutely bizarre** to represent Sharon's reasoning as starting with Q₂
- Sharon starts with Q₁!

Back to the Students

- (P_1) There will be a class test one day next week, but the students won't know what day the test is happening until a few minutes before it happens
- (P_2) The Teacher has promised her students that there will be a class test one day next week, but the students won't know what day the test is happening until a few minutes before it happens
- When telling the story of the unexpected exams, I have to say P_2
 - But when representing the students' reasoning, shouldn't we start with P_1 (just like we did with Sharon)?
 - If so, Quine's solution won't work
 - No! We cannot start the students' reasoning with P_1

Starting with P_1

- (1) Suppose P_1
 - (2) The exam has not occurred by Thursday night
 - (3) Therefore, the exam must occur on Friday
 - (4) **Therefore, the students can figure out that the exam will occur on Friday**
 - (5) Therefore not- P_1
- (6) Therefore, not- P_1 (there will not be a surprise exam)
 - We have given **no justification** for (4)!
 - None of the premises tell us anything about what the students know, or what they can figure out

Starting with knowledge of P_1

- (1) Suppose that the students **know that** P_1
 - (2) Therefore P_1
 - (3) The exam has not occurred by Thursday night
 - (4) Therefore, the exam must occur on Friday
 - (5) Therefore, the students can figure out that the exam will occur on Friday
 - (6) Therefore, not- P_1 (there will not be a surprise exam)
- (7) **Therefore not- P_1**
 - The move from (1) to (2) is justified by the fact that knowledge implies truth
 - The move from (4) to (5) is justified because the students can deduce that the exam will occur on Friday from things they already know
 - But we can't infer (7)

Starting with knowledge of P_1

- (1) Suppose that the students **know that** P_1
 - (2) Therefore P_1
 - (3) The exam has not occurred by Thursday night
 - (4) Therefore, the exam must occur on Friday
 - (5) Therefore, the students can figure out that the exam will occur on Friday
 - (6) Therefore, not- P_1 (there will not be a surprise exam)
- (7') Therefore the students **do not know that** P_1
 - The move from (1) to (2) is justified by the fact that knowledge implies truth
 - The move from (4) to (5) is justified because the students can deduce that the exam will occur on Friday from things they already know
 - Now all we can infer is (7')

The Students *can* be Surprised!

- The beauty of this response to the paradox is that we can now see that The Teacher **definitely can** surprise her students with an exam
 - The students cannot **know** P_1 , i.e. they cannot know that there will be an unexpected exam
 - So they cannot have any **rational expectations** about when the exam will be!
- But there is still a bit of a sting in the tail of this paradox...

The Paradox Strikes Back

- If the students are smart, then they should be able to go through all of the reasoning that we just went through, and discover that The Teacher really can surprise them with an exam, if she wants
- And that is exactly what The Teacher has promised them she will do
- So The Teacher has promised to do something which the students know she can do, plus we can imagine that in general she is a very honest, reliable person
- Shouldn't that be enough to let the students **know** that there will be a surprise exam!?!?

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Getting rid of Epistemology

- I have so far framed the Unexpected Exam Paradox in **epistemological** terms
- The paradox is about an exam which no one can **know** is happening until just before it happens
- But many philosophers, e.g. Shaw (1958), have pointed out that we can reformulate the paradox in **logical**, not epistemological, terms

A New Premise

(P_3) One day next week there will be a test, and on no day before the end of this week will it be possible to deduce, from this announcement and the fact that the test has not been given yet, what day the test will occur

- P_3 is self-contradictory
 - If it gets to Thursday and the students have not yet had the exam, then they could use that fact along with the first conjunct of P_3 to deduce that the exam will be on Friday
- So if The Teacher announced P_3 , she would have contradicted herself

A Veridical Paradox (for real this time!)

- But this really is a **veridical paradox**
- To see this, compare (P_3) with:
 - (A) Grass is green, but you cannot deduce that grass is green from A
- (A) simply cannot be true
 - The first conjunct is true
 - But obviously, you *can* deduce from (A) that grass is green

A Veridical Paradox (for real this time!)

- The same goes for (P_3)
 - (P_3) One day next week there will be a test, and on no day before the end of this week will it be possible to deduce, from this announcement and the fact that the test has not been given yet, what day the test will occur
- (P_3) cannot be true, because if it gets to Thursday you will be able to deduce that the test will be on Friday
- But what about the earlier problem: can't The Teacher **make** P_3 true just by setting the exam on Tuesday?
- No!
- The Teacher **can** make the first conjunct of (P_3) true by setting a test, but not (P_3) as a whole

Next Week

- We will be looking at Zeno's Paradoxes
- Required Reading:
 - Sainsbury's *Paradoxes*, Chapter 1
- Recommended further reading
 - For some helpful background, including discussion of Zeno's mentor, Parmenides, you may also want to look at Chapters 3 and 4 of Sorensen's (2003) *A Brief History of the Paradox* (OUP)

References

- Quine, WVO (1953) 'On a supposed paradox', reprinted as 'On a supposed antinomy' in Quine's *The Ways of Paradox and Other Essays* (1976), pp. 19–21
- Scriven, M (1951) 'Paradoxical Announcements', *Mind* 60: 403–7
- Shaw, R (1958) 'The Paradox of the Unexpected Examination', *Mind* 67: 382–4
- For a comprehensive overview: Margalit and Bar-Hillel (1983) 'Expecting the unexpected', *Philosophia* 13: 263–88.
(Unless you are **really** interested in the history of the paradox, you might want to skip to p. 278!)

Thanks again to Martin Austwick and Daley Walton for allowing me to use their image of a man shaving <https://thesoundoftheladies.bandcamp.com>