

Paradoxes

Lecture One

What is a Paradox?

Rob Trueman
rob.trueman@york.ac.uk

University of York

What is a Paradox?

Preliminaries

What is a Paradox?

Some Examples

Are Paradoxes Arguments?

Why Should We Care?

Why Study Paradoxes?

- The history of philosophy is a history of paradoxes!
- Many of the best philosophical ideas started off life as reactions to paradoxes
- Studying paradoxes is a way of engaging with some of the deepest, and also some of the *oldest*, philosophical issues there are
- What is more, modern logic provides us with new ways of engaging with these old puzzles

Teaching

- Contact Hours
 - 9 × 1 hour lectures (Tuesday 15:00–16:00)
 - 4 × 1.5 hour seminars (Weeks 4, 6, 7 and 9 — check your timetable!)
 - Weekly Office Hours (Monday 15:30–16:30 & Thursday 12:30–13:30)
- Procedural Requirements
 - Attend lectures
 - Complete all required reading
 - Attend, and fully participate in, seminars

Textbooks

- **The main textbook:**
 - Sainsbury, M (2009) *Paradoxes*, 3rd edition, CUP

- **Some more helpful texts:**
 - Clark, M (2007) *Paradoxes from A to Z*, 2nd edition, Routledge
 - Sorensen, R (2003) *A Brief History of the Paradox*, OUP

The Reading List

- In addition to these texts, there is a full Reading List on the VLE site
- Readings marked **Essential** must be read in preparation for this module
- Readings marked **Recommended** would be good to read to get a fuller understanding of the material
- Readings marked **Background** are usually more advanced texts, and you only need to read them if you really want a deeper understanding

Seminars

- Some of the items on the Reading List are marked as **Seminar Reading**
- You **must** read these **before** each seminar
- Before each seminar, you will be provide with some questions to think about while doing your reading
- You should bring **written** answers to these questions to each seminar

Assessment

- **Summative Assessment**

- 60 minute exam
- Summer Assessment Period
- Worth 100% of the module (10 credits)

- **Formative Assessment**

- Practice exam answer
- Due noon, Monday Week 8
- Questions will be made available on the VLE

What is a Paradox?

Preliminaries

What is a Paradox?

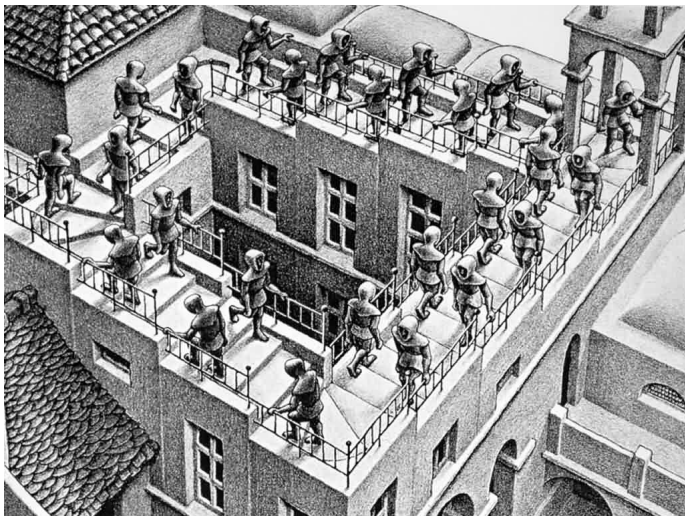
Some Examples

Are Paradoxes Arguments?

Why Should We Care?

Paradoxes (1): What is a Paradox?

└ What is a Paradox?



Paradoxes (1): What is a Paradox?

└─What is a Paradox?



Etymology

Para

Contrary to

Doxa

Received Opinion
/ Belief

Quine's Definition

May we say in general, then, that a paradox is just any conclusion that at first sounds absurd but that has an argument to sustain it? In the end I think this account stands up pretty well. But it leaves much unsaid.

Quine (1961), p. 1



Sainsbury's Definition

*an apparently unacceptable conclusion derived by
apparently acceptable reasoning from apparently
acceptable premises*

Paradoxes, p. 1

Sainsbury's Definition

*an **apparently** unacceptable conclusion derived by **apparently** acceptable reasoning from **apparently** acceptable premises*

Paradoxes, p. 1

Something's Gotta Give

An unacceptable conclusion derived by acceptable reasoning from acceptable premises

Something's Gotta Give

~~An unacceptable conclusion derived by acceptable reasoning from acceptable premises~~

Something's Gotta Give

~~An unacceptable conclusion derived by acceptable reasoning from acceptable premises~~

- Reject one (or more) of the premises
- Reject the reasoning
- Accept the conclusion after all

Quine's Categorisation

~~An unacceptable conclusion derived by acceptable reasoning from acceptable premises~~

- Reject one (or more) of the premises — Falsidical
- Reject the reasoning — Falsidical
- Accept the conclusion after all — Veridical

Lycan's Modification

~~An unacceptable conclusion derived by acceptable reasoning from acceptable premises~~

- Reject one (or more) of the premises — Premise-flawed
- Reject the reasoning — Falsidical
- Accept the conclusion after all — Veridical

Another Modification

~~An unacceptable conclusion derived by acceptable reasoning from acceptable premises~~

- Reject one (or more) of the premises — Premise-flawed
- Reject the reasoning — Fallacious
- Accept the conclusion after all — Veridical

Antinomies

~~An unacceptable conclusion derived by acceptable reasoning from acceptable premises~~

- Reject one (or more) of the premises — Premise-flawed
- Reject the reasoning — Fallacious
- Accept the conclusion after all — Veridical

- Sometimes we do not know how to deal with a paradox
- Quine called paradoxes like these **antinomies**

What is a Paradox?

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Why Should We Care?

The Barber Paradox



The Barber Paradox

- The Barber shaves **only** the people in his village who do not shave themselves
- The Barber shaves **all** the people in his village who do not shave themselves
- Does The Barber shave himself?



The Barber Paradox



- If The Barber shaves himself, then he **does not** shave himself
- If The Barber does not shave himself, then he **does** shave himself
- So The Barber both does and does not shave himself!

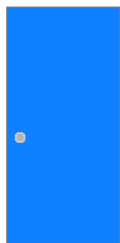
What Sort of Paradox is This?

- Premise-flawed
 - One of the premises is false
- Fallacious
 - The reasoning is not valid
- Veridical
 - The conclusion is true after all

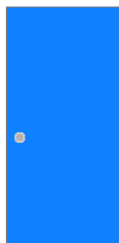
What Sort of Paradox is This?

- This paradox is **premise-flawed**
- The Barber Paradox just shows that it is impossible for there to be a barber who shaves all and only the people in his village who do not shave themselves
- This might be surprising, but sometimes contradictions can be well hidden
- And of course, it is not impossible for there to be a barber who **adopts the policy** of shaving all and only those who do not shave themselves
 - That policy is inconsistent, but people adopt inconsistent policies all the time!

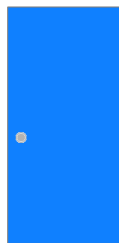
The Monty Hall Paradox



Door 1



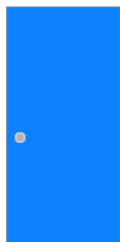
Door 2



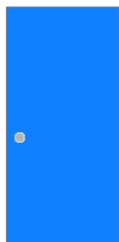
Door 3

There is a car behind one of these doors

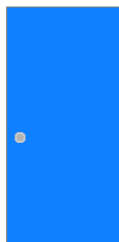
The Monty Hall Paradox



Door 1



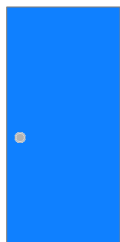
Door 2



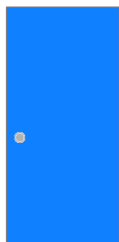
Door 3

I am a gameshow host, and I know where the car is

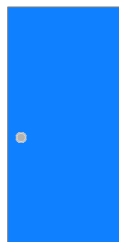
The Monty Hall Paradox



Door 1



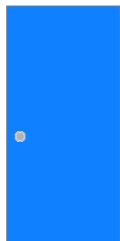
Door 2



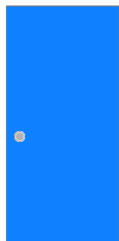
Door 3

You are a contestant, and you do not know where the car is

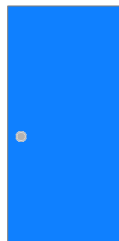
The Monty Hall Paradox



Door 1



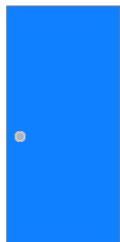
Door 2



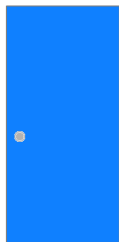
Door 3

I tell you to pick any door

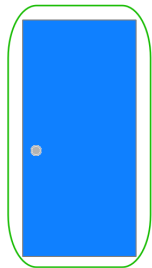
The Monty Hall Paradox



Door 1



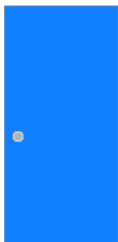
Door 2



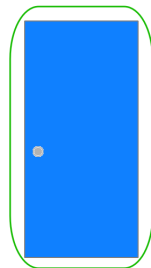
Door 3

You pick Door 3

The Monty Hall Paradox



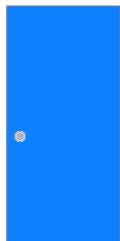
Door 1



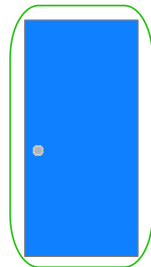
Door 3

I then reveal that the car is not behind Door 2

The Monty Hall Paradox



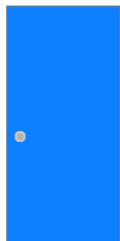
Door 1



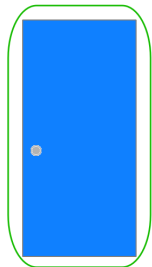
Door 3

Now I ask you if you would like to swap doors

The Monty Hall Paradox



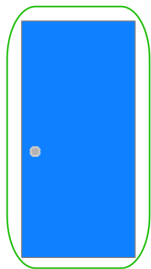
Door 1



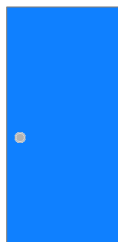
Door 3

Question: Should you swap doors?

The Monty Hall Paradox



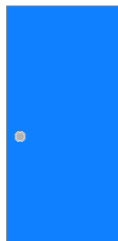
Door 1



Door 3

You should swap doors!

The Monty Hall Paradox



Door 3

You will double your chances of winning!!

The Monty Hall Paradox

- **Suppose the car is behind Door 1**
 - In that case, the I must open Door 2
 - And so if you swap, you will end up with Door 1, and will get the car
- **Suppose the car is behind Door 2**
 - In that case, I must open Door 1
 - And so if you swap, you will end up with Door 2, and will get the car
- **Suppose the car is behind Door 3**
 - In that case I will choose at random whether to open Door 1 or Door 2
 - And whichever I choose, if you swap, you will lose the car

The Monty Hall Paradox

- So if you swap, you will win in 2 out of the 3 possible scenarios
- So if you swaps, your odds of winning are $2/3$
- But your initial odds of winning, when you chose Door 1 in the first place, were $1/3$
- So you double your chances of winning by swapping!

What Sort of Paradox is This?

- This paradox is **veridical**
- You really will double your odds of winning if you swap doors
- This can be rigorously proven in probability theory
- It can also be checked experimentally
 - If we played the game 3,000 times, you would win roughly 2,000 times if you swapped, and only 1,000 times if you didn't

$$2 = 1$$

$$\text{Let } x = y \neq 0$$

$$x^2 = xy \quad (\text{Since } x = y)$$

$$x^2 - y^2 = xy - y^2 \quad (\text{Subtract } y^2 \text{ from both sides})$$

$$(x + y)(x - y) = y(x - y) \quad (\text{Re-write both sides})$$

$$x + y = y \quad (\text{Divide both sides by } x - y)$$

$$2y = y \quad (\text{Since } x = y)$$

$$\therefore 2 = 1 \quad (\text{Since } y \text{ is any non-0 number})$$

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$$x + y = y \quad (\text{Divide both sides by } x - y)$$

$$2y = y \quad (\text{Since } x = y)$$

$$\therefore 2 = 1 \quad (\text{Since } y \text{ is any non-0 number})$$

The Fallacy in the Proof

- This paradox is **fallacious**
- If $x = y$, then $x - y = 0$, and you cannot divide by 0!
- People sometimes say that a number divided by 0 is infinity ($n \div 0 = \infty$), but that is wrong
- If it were right, we could prove that $2=1$ really quickly:

$$2 \div 0 = \infty$$

$$1 \div 0 = \infty$$

$$2 \div 0 = 1 \div 0$$

$$2 = 1$$

Grelling's Paradox

- Some adjectives apply to themselves
 - 'Short' is short
 - 'English' is English
 - 'Adjectival' is adjectival
- Other adjectives do not apply to themselves
 - 'Long' is not long
 - 'Monosyllabic' is not monosyllabic
 - 'French' is not French
- Call the adjectives that do not apply to themselves **heterological**
 - x is heterological iff x does not apply to x

Grelling's Paradox

- Is 'heterological' heterological?
 - x is heterological iff x does not apply to x
- Suppose that 'heterological' is heterological
 - So 'heterological' applies to itself
 - But in that case, 'heterological' **is not** heterological!
- Now suppose that 'heterological' is not heterological
 - So 'heterological' does not apply to itself
 - But in that case, 'heterological' **is** heterological!
- So 'heterological' **is** heterological iff 'heterological' **is not** heterological!

What Sort of Paradox is This?

- Premise-flawed?
- We only used one premise:
 - ‘Heterological’ applies to ‘heterological’ iff ‘heterological’ is heterological
- This is just a particular instance of the following general principle:
 - ‘ F ’ applies to x iff Fx
- This general principle seems to be built into what we mean when we talk about a predicate *applying* to an object

What Sort of Paradox is This?

- Fallacious?
- All of the reasoning is classically valid
 - All of the reasoning is licensed by the standard rules of validity we normally think are totally unproblematic
- If we want to say that Grelling's Paradox is fallacious, then we have to somehow modify classical logic

What Sort of Paradox is This?

- Veridical?
- The conclusion of Grelling's Paradox is a contradiction
- So if we want to say that it is veridical, then we must accept that some contradictions are true!

What Sort of Paradox is This?

- None of these options seem very good
- So Grelling's Paradox is an **antinomy!**

Antinomies

it is [antinomies] that bring on the crises in thought. An antinomy produces a self-contradiction by accepted ways of reasoning. It establishes that some tacit and trusted pattern of reasoning must be made explicit and henceforward be avoided or revised.

Quine (1961), p. 5

Antinomies

A veridical paradox packs a surprise, but the surprise quickly dissipates itself as we ponder the proof. A falsidical [i.e. premise-flawed or fallacious] paradox packs a surprise, but it is seen as a false alarm when we solve the underlying [mistake]. An antinomy, however, packs a surprise that can be accommodated by nothing less than a repudiation of part of our conceptual heritage.

Quine (1961), p. 9

Antinomies

*One man's antinomy is another man's falsidical paradox,
give or take a couple of thousand years.*

Quine (1961), p. 9

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Are Paradoxes Arguments?

Why Should We Care?

What is an Argument?

- Consider the following **argument**

$$P_1, P_2, \dots, \therefore C$$

- As Lycan (2010) points out, to say that this argument is valid is just to say that the following **set of propositions** is inconsistent:

$$\{P_1, P_2, \dots, \neg C\}$$

- It is up to us which propositions we choose to think of as premises, and which we choose to negate to make the conclusion

$$\neg C, P_1, \dots, \therefore \neg P_2$$

Lycan's Objection to Quine's Categorisation

- Whether we treat a paradox as premise-flawed or veridical just depends on how we choose to set the argument out
- Imagine we have a paradox:

$$P_1, P_2, \dots, \therefore C$$

- And suppose that we resolve the paradox by declaring that P_2 is false (despite initial appearances)
- We would say that we have a **premise-flawed** paradox
- But we could just as well write the paradox as:

$$\neg C, P_1, \dots, \therefore \neg P_2$$

- Now we would say that the paradox is **veridical**, since its conclusion, $\neg P_2$, is true (despite initial appearances)

Lycan's Account

a paradox [...] is an inconsistent set of propositions, each of which is very plausible.

Lycan (2010), p. 618

- Since paradoxes have to be **inconsistent** sets on Lycan's view, there are no fallacious paradoxes
- A 'fallacious paradox' is really just a set of sentences which **appears** paradoxical, but in fact isn't
- There is no real problem with that, but it seems a needlessly fussy way of talking
- A paradox is an **apparently** inconsistent set of propositions, each of which is very plausible

Quine's Categorisation as Conventional

- Lycan has not shown that we should stop talking about 'veridical' and 'premise-flawed' paradoxes
- He has just shown that whether a paradox counts as veridical or premise-flawed is really just a matter of how we have chosen to represent it
- But whether a paradox is an **antinomy** is not conventional in this way
- To call a paradox an antinomy is to say that we just do not know what to do with it!

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Galileo's Paradox

- Surely there are more whole numbers than there are **even** whole numbers

1	—	
2	—	2
3	—	
4	—	4
...		...

Galileo's Paradox

- But we can find a way of pairing an even number with each and every number

1	—	2
2	—	4
3	—	6
4	—	8
...		...

- So there are just as many even numbers as there are numbers!

Galileo's Paradox

- (1) There are more whole numbers than even whole numbers
 - (2) There is a way of pairing off each whole number with an even whole number
 - (3) If you can pair off each whole number with an even whole number, then there are (at least) as many even whole numbers as there are whole numbers
- This was once an antinomy, but now mathematicians just accept that (1) is false
 - They **use** the fact that (1) is false to prove all sorts of important things about infinity
 - A set is infinite iff there is a 1-1 correspondence between that set and one of its proper subsets

Paradox and the Philosophical Experience

- Philosophy is all about being confronted with tensions in our thought
- Maybe not all of these tensions deserve to be called 'antinomies', or even just 'paradoxes'
- But antinomies are just the extreme end of the philosophical experience
- Understanding this extreme end will help you to understand philosophy itself

Next Week

- We will be looking at the Unexpected Exam
- Required Reading:
 - Sainsbury's *Paradoxes*, Chapter 5, §§5.2–5.3
- Recommended Further Reading:
 - Quine, WVO (1953) 'On a supposed paradox', reprinted as 'On a supposed antinomy' in Quine (1976), pp. 19–21

References

- Lycan, G (2010) 'What, exactly, is a paradox?', *Analysis* 70: 1–8
- Quine, WVO (1961) 'The ways of paradox', reprinted in Quine (1976) pp. 1–18
- ——— (1976) *The Ways of Paradox and Other Essays* (Harvard University Press: Cambridge, MA)
- Thanks to Martin Austwick and Daley Walton for allowing me to use the image of a man shaving, which was an illustration by Daley for Martin's song *Wash the sleep from my eyes, for one day I'll have to awake*
<https://thesoundoftheladies.bandcamp.com>