Intermediate Logic
Lecture One

Truth-Functional Logic

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Truth-Functional Logic

A Module of Two Halves

- This module is split into two halves: an Autumn Half, and a Spring Half
- In the Autumn Half, we are going to be focussing on classical logic
- In the Spring Half, we are going to look at some non-classical logics

The Aims of the Autumn Half

- In the Autumn Half, we are going to build on what you have already learnt in *Reason & Argument*
- In *Reason & Argument*, you learnt how to formalise some natural language sentences and arguments
- You also learnt how to draw up truth-tables to test the validity of some arguments

The Aims of the Autumn Half

- In this module, you will learn how to present rigorous, *formal proofs*
  - This will give you a whole new way of showing that an argument is valid
- You will also learn how to present *interpretations* for predicate logic
  - This will give you a systematic way of showing that an argument in predicate logic is *invalid*

Why Study Intermediate Logic?

- **A boringly practical answer:**
  - Lots of philosophers just assume that their audience will be familiar with a fair bit of logic; if you want to understand them, you need to know your logic
- **An answer that is a bit more interesting:**
  - Although you probably won't write out too many formal proofs in your everyday life, learning how to prove things in a completely rigorous way will help you to think more carefully day to day

Why Study Intermediate Logic?

- **The best answer:**
  - It is remarkable that formal proofs are so much as *possible*
  - A formal proof is a proof whose correctness can be checked *mechanically*: you do not need to be smart or have any flashes of insight
  - That is amazing: in most spheres of life, there is no mechanical method for checking whether what we did was correct!
  - In fact, for a long time philosophers thought that hardly any reasoning could be dealt with according to formal rules
  - We now know that *lots* of it can!

Contact Hours

- Weekly lectures (Tuesday 13:00–15:00)
- Weekly Office Hours (Wednesdays 10:30–11:30 & Thursdays 15:30–16:30)
- Logic Café (Tuesdays 15:00–16:00, Derwent Café)

The Textbook

- The textbook we are using for this module is:
  forallχ: York Edition
- You can download a copy of this textbook from the VLE page for the module
- *forallχ* is an "open source" textbook; it was originally written by P.D. Magnus, it was then modified by Tim Button, and now it has been further modified for this module by me
- Since this textbook is open source, we can make changes to it at **any time**; so if you spot any mistakes, or if there is anything you wish were covered but isn't, let me know!

The Lectures

- These lectures are probably better described as **classes**
  - Some of the class will be taken up with me explaining the crucial ideas
  - But lots, sometimes even most, of the class will be spent completing exercises together

- I should also make clear now that the explanations that I will offer in this class will sometimes be rough and ready, designed to convey the key idea rather than all of the details

- The details are all covered in the textbook, and so it is essential that you complete **all** of the specified reading **before** each class

(and it would be a good idea to **re-read** it all after the class too!)

Exercises

- We will go through lots of exercises in these classes, but there are lots **more** exercises to do in *forall*χ
- It is **essential** that you do **all** of those exercises!!!
- It is only by doing those exercises that you will really learn how to use formal logic
- There is a Solutions Booklet on the VLE page, which has solutions for all of the exercises in *forall*χ

Peer Assisted Learning

- Intermedaite Logic is supported by Peer Assisted Learning (PAL) sessions
- In a PAL session, groups of students currently taking Intermediate Logic meet with 3rd year students who took the module last year
- These are a *great* opportunity for guided practice
- **Thursdays, 2pm-3pm, at G/N/013**
- Each week, you can sign up for the PAL session at: https://goo.gl/forms/7y4iXX6D83ftWwJG3

Summative Assessment

- The Summative Assessment for the Autumn Half of this module will be a 1 hour exam, held in the Spring Exam period
  - This exam is worth 50% of the full module, and 100% of the short module
- In this exam, you will be tested on your ability to present proofs and counter-interpretations
- You can find some past exams on the VLE which will give you a good sense of what to expect

Formative Assessment

- There will be three opportunities for Formative Assessment: the first will be due Monday Week 5; the second will be due Monday Week 7; the third will be due Monday Week 10
- These assessments will take the form of worksheets, which will be posted on the VLE
- I strongly recommend that you take these opportunities for feedback!

High Risk, High Reward

- With hard work, it is possible to do *very* well in the exam
  - Last year, a number of people got over 90%, and some even got 100%!
- However, failing this exam is also a *real* possibility
- **To do well, it is absolutely essential that you:**
  - Keep up with all of the textbook readings
  - Do all of the exercises in the textbook
  - Complete all of the formative assessments
  - Attend all of the lectures
  - Make good use of office hours, etc.

Truth-Functional Logic

Validity and Soundness

- An argument is **valid** if and only if it is impossible for all of its premises to be true and its conclusion false
  - Sharon studies archaeology
  - If Sharon studies arcaheology, then she tells Rob a lot about old pots
  - So Sharon tells Rob a lot about old pots

- An argument is **sound** if and only if it is valid and all of its premises are true

- Sound arguments are even better than valid ones, but logic is just about sorting the *valid* arguments from the *invalid* ones

Valid in Virtue of Form

Sharon studies archaeology

If Sharon studies arcaheology, then she tells Rob a lot about old pots

So Sharon tells Rob a lot about old pots

Valid in Virtue of Form

It is raining outside

If it is raining outside, then Simon is miserable

So Simon is miserable

Valid in Virtue of Form

A

If A, then B

So B

Any argument with this form is valid!

Atomic Sentences

- **Truth-Functional Logic** (TFL) is an artificial language which allows us to symbolise many arguments in a way that reveals their form

  (In *Reason and Argument*, you called TFL 'Propositional Logic')

- The basic building blocks in TFL are **atomic sentences**; all other sentences are ultimately built out of these atoms

- We use capital letters as atomic sentences:

  $A, B, C, \ldots, Z, \; A_1, B_1, \ldots, \; A_2, B_2, \ldots$

Complex Sentences

Connective	TFL Symbol	English Reading	*R&A*
Negation	$\neg\mathcal{A}$	It is not the case that $\mathcal{A}$	$\sim\mathcal{A}$
Conjunction	$(\mathcal{A} \wedge \mathcal{B})$	$\mathcal{A}$ and $\mathcal{B}$	$(\mathcal{A} \& \mathcal{B})$
Disjunction	$(\mathcal{A} \vee \mathcal{B})$	$\mathcal{A}$ or $\mathcal{B}$	$(\mathcal{A} \vee B)$
Conditional	$(\mathcal{A} \rightarrow \mathcal{B})$	If $\mathcal{A}$ then $\mathcal{B}$	$(\mathcal{A} \supset \mathcal{B})$
Biconditional	$(\mathcal{A} \leftrightarrow \mathcal{B})$	$\mathcal{A}$ if and only if $\mathcal{B}$ $\mathcal{A}$ iff $\mathcal{B}$	$(\mathcal{A} \equiv \mathcal{B})$

Examples

A: Daniel is sleeping

B: Daniel is snoring

C: Simon is irritated

Daniel is sleeping and Daniel is snoring $\Rightarrow A \wedge B$

Daniel is snoring or he isn't sleeping $\Rightarrow B \vee \neg A$

If Daniel is sleeping, then Daniel is snoring and Simon is irritated $\Rightarrow A \to (B \wedge C)$

Exercises

Symbolise each English sentence in TFL:

M: Those creatures are men in suits

C: Those creatures are chimpanzees

G: Those creatures are gorillas

1. Those creatures are not men in suits.
2. Those creatures are either gorillas or chimpanzees.
3. Those creatures are neither gorillas nor chimpanzees.
4. If those creatures are chimpanzees, then they are neither gorillas nor men in suits.
5. Those creatures are chimpanzees, unless they are either gorillas or men in suits.

Truth-Functional Logic

Truth-Functionality

- All of the connectives in TFL are **truth-functional**
  - That is why we call it *truth-functional logic*
- When you combine some sentences with a TFL connective, the truth-value of the resulting sentence is completely determined by the truth-values of the original sentences

  Example: $\neg\mathscr{A}$ is true iff $\mathscr{A}$ is true, and $\neg\mathscr{A}$ is false iff $\mathscr{A}$ is true
- We can use **truth tables** to represent how the truth-values of complex sentences depend on the truth-values of simpler sentences

The Characteristic Truth Tables

$\mathcal{A}$	$\neg\mathcal{A}$
T	F
F	T

The Characteristic Truth Tables

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \wedge \mathcal{B}$
T	T	T
T	F	F
F	T	F
F	F	F

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \vee \mathcal{B}$
T	T	T
T	F	T
F	T	T
F	F	F

The Characteristic Truth Tables

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \rightarrow \mathcal{B}$
T	T	T
T	F	F
F	T	T
F	F	T

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \leftrightarrow \mathcal{B}$
T	T	T
T	F	F
F	T	F
F	F	T

An Example

P	Q	$(P$	$\vee$	$Q)$	$\leftrightarrow$	P
T	T	T	T	T	**T**	T
T	F	T	T	F	**T**	T
F	T	F	T	T	**F**	F
F	F	F	F	F	**T**	F

- The truth-values under the biconditional are the different truth-values that the *whole* sentence has on each line
- The biconditional is the **main logical connective** in this sentence, meaning that it was the last connective used in its construction

The Main Logical Connective

- Here's how you find the main logical connective:
  - Make sure you have included **all** the brackets in the sentence
  - Check if the first symbol in the sentence is a '$\neg$'; if it is, then that '$\neg$' is the main logical connective
  - Otherwise, start counting brackets — open brackets are worth $+1$, close brackets are worth -1. The first connective you hit when the count is exactly 1 *which is not a* '$\neg$' is the main logical connective

- **Examples:**
  - $((\neg(A \wedge B) \vee (P \rightarrow Q)) \leftrightarrow (P \vee A))$
  - $((((Q \vee R) \rightarrow S) \wedge (P \vee Q)) \leftrightarrow \neg A)$

Exercises

Offer truth tables for each of the following:

1. $\neg(A \vee \neg A)$
2. $(A \rightarrow B) \vee (B \rightarrow A)$
3. $\neg((C \vee A) \leftrightarrow B)$
4. $(\neg(A \vee B) \wedge (\neg C \rightarrow B)) \wedge \neg(C \wedge \neg A)$

Truth-Functional Logic

Validity and Possible Worlds

- An argument is valid iff it is impossible for all of its premises to be true and its conclusion false
- **Introducing Possible World Speak:**
  - If it is possible for $\mathcal{A}$ to be true, then $\mathcal{A}$ is true *in some possible world*
  - If it is impossible for $\mathcal{A}$ to be true, then $\mathcal{A}$ is not true *in any possible world*
- An argument is valid if there is no possible world in which all of its premises are true and its conclusion is false

Valuations

- Possible world speak is really great, but it isn't very *precise*; so our definition of validity isn't very precise either
- Happily, we can give a more rigorous definition of a notion of validity designed for TFL
- We just need to replace the idea of a possible world with the idea of a **valuation**
  - A valuation is any assignment of truth-values to atomic sentences of TFL
  - Each line of a truth table is a valuation; a whole truth table covers all the possible valuations of the atoms listed at the top of the table

Tautological Entailment

- The sentences $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ **tautologically entail** sentence $\mathcal{C}$ iff there is no valuation on which all of $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are true and $\mathcal{C}$ is false

- We will use '$\vDash$' (the *double-turnstile*) to express tautological entailment
  - $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ tautologically entail $\mathcal{C}$
  - $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n \vDash \mathcal{C}$

Testing for Tautological Entailment

- To test whether a collection of premises tautologically entail a given conclusion, draw up a truth table for all of those sentences together
- Then check if there is any line on which all of the premises are true and the conclusion is false
- If so, then the premises do not tautologically entail the conclusion
- If not, then the premises do tautologically entail the conclusion

Testing for Tautological Entailment

$\neg A,\ A \vee B \ \vDash\ B$

A	B	$\neg$	A	A	$\vee$	B	B
T	T	**F**	T	T	**T**	T	**T**
T	F	**F**	T	T	**T**	F	**F**
F	T	**T**	F	F	**T**	T	**T**
F	F	**T**	F	F	**F**	F	**F**

Exercises

Use truth tables to determine whether the following claims are true

1. $A \vee (B \rightarrow A) \vDash \neg A \rightarrow \neg B$
2. $A \rightarrow B,\ \neg A \vDash \neg B$
3. $\neg A \vee B,\ \neg(B \wedge \neg A) \vDash A \leftrightarrow B$
4. $A \leftrightarrow B,\ B \vee C,\ \neg A \vDash \neg\neg C$

Truth-Functional Logic

Object-Language versus Meta-Language

- In this lecture, we have been talking a lot *about* TFL, but we haven't been speaking *in* TFL
- We have used plain English to talk about TFL
- Logicians mark this distinction by saying that TFL is our **object-language**, and English is our **meta-language**
- It is absolutely crucial to bear in mind this distinction between object-language and meta-language!

Use versus Mention

- We can't really *use* any of the sentences of TFL in English
  - '$A \wedge B$' is a perfeclty good sentence of TFL, but that string of symbols is not English

- What we can do is *mention* the sentences and symbols of TFL in English — that is, we can talk *about* them

- We indicate that this is what we are doing by writing them within quotation marks
  - × $A \wedge B$ and snow is white
  - ✓ '$A \wedge B$' is true if and only if 'A' is true and 'B' is true

Swash Fonts

- Quotation marks are great for talking about *particular* sentences from TFL, but what should we do when we want to talk about TFL sentences in general?
- **Answer:** We use fancy swash fonts to generalise over sentences
  - $\mathcal{A} \wedge \mathcal{B}$ is true iff $\mathcal{A}$ is true and $\mathcal{B}$ is true
  - Take any two TFL sentences, $\mathcal{A}$ and $\mathcal{B}$, and write them either side of the symbol '$\wedge$'; the resulting sentence is true iff $\mathcal{A}$ is true and $\mathcal{B}$ is true

'$\vDash$' is in the Meta-Language

- It is **very** important to realise that our symbol for tautological entailment, '$\vDash$', belongs to our meta-language, not to the object-language TFL
- When we say that some TFL sentences tautologically entail another TFL sentence, we are talking *about* TFL
- So TFL is our object-language, and the meta-language is (a slightly augmented version of) English

'$\vDash$' versus '$\to$'

- It is especially important not to confuse '$\vDash$' with '$\to$'
- '$\to$' is a TFL connective, with the following truth table:

$\mathcal{A}$	$\mathcal{B}$	$\mathcal{A} \to \mathcal{B}$
T	T	T
T	F	F
F	T	T
F	F	T

- '$\vDash$' does not belong to TFL, but to our meta-language, and we use it to express tautological entailment

But an Important Relationship...

- Some sentences are true on *every* line of the truth table
- We call these sentences **tautologies**, and we can indicate that $\mathcal{A}$ is a tautology simply by writing: $\vDash \mathcal{A}$
- An important relationship between '$\vDash$' and '$\rightarrow$':
  - $\mathcal{A} \vDash \mathcal{B}$ iff $\vDash \mathcal{A} \rightarrow \mathcal{B}$
  - $\mathcal{A}$ tautologically entails $\mathcal{B}$ if and only if the conditional $\mathcal{A} \rightarrow \mathcal{B}$ is a tautology
- This is an important relationship, but it doesn't change the fact that '$\vDash$' and '$\rightarrow$' are different symbols, which mean different things and belong to different languages

For Next Week

- Read §§15–16 of *forallχ*
- Attempt **all** of the exercises!