Theories 4

Rob Trueman
rt295@cam.ac.uk

11/02/13

1 The correspondence

- Between 1/10/1895 and 7/11/1903, Frege and Hilbert exchanged a number of letters
- They discussed the following topics:
  - The idea that axioms implicitly defining the primitives they contain
  - The very possibility of proving consistency
  - The idea that (at least in maths) consistency implies truth

2 Hilbert

- Hilbert thought of consistency in primarily syntactic terms
- There was also an allied notion of property-consistency
- If there are n non-logical primitives in a set of sentences, then that set will define an n-place complex property
- We say that a complex property defined in this way by some axioms is consistent iff some series of concepts/relations could satisfy it
- Hilbert's syntactic consistency proofs obviously establish property-consistency
- And so long as the deductive system is complete, property-consistency entails syntactic consistency

3 Frege

- Frege had a division between language (strings of symbols), reference (the things bit of language can be used to talk about), and sense (how certain bits of language present the things that they refer to)
- A proposition is the sense of a sentence

- For Frege, logical relations don't really (or primarily) hold between sentences, but between propositions

- So his notion of logical consequence (Frege-consequence) is not syntactic

- For Frege, a good formal deductive system is such that for any sentences ϕ and ψ, ψ is derivable from ϕ iff the proposition expressed by ψ is a Frege-consequence of the proposition expressed by ϕ

- But, for Frege it is important that syntactic derivability and Frege-consequence can come apart in a particular way

4 Understanding the debate

- Hilbert showed that the axioms of hyperbolic geometry were syntactically consistent, not Frege-consistent

- Frege also doubted that we could *ever* prove some axioms to be Frege-consistent. We can prove that axioms are syntactically consistent, but that just doesn't translate into Frege-consistency

- Frege insists that the only way to show that some concepts/relations are mutually consistent is by finding something which satisfies those concepts

- This gels nicely with his idea that axioms define second-level concepts

- The way that Hilbert shows that the second-level relation defined by the axioms of hyperbolic geometry, H, is consistent is precisely by finding some first-level concepts/relations which satisfy H: in particular, he finds some concepts/relations of real numbers which satisfy H