

Paradoxes Bonus Slides

The Liar Paradox: Ungrounded Sentences

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The Liar Paradox

Kripke's Solution: Truth-Value Gaps

An Objection to Kripke's Solution

Kripke's Solution

- Kripke offered an alternative solution to the Liar Paradox, which was not quite so restrictive as Tarski's
- In Kripke's system, languages are allowed to be semantically closed
- And there is just one notion of truth
- This gets us around both of the problems we looked at for Tarski's system

Explaining Truth to the Uninitiated

- Suppose that someone was speaking a language, L , which talks only about the world, but not about truth
- In this language you can say things like 'Snow is white' and 'Grass is green'
- Suppose we wanted to explain to this person what 'is true' means

Explaining Truth to the Uninitiated

We may say that we are entitled to assert (or deny) of any sentence that it is true precisely under the circumstances when we can assert (or deny) the sentence itself.

(Kripke 1975, p. 700)

Explaining Truth to the Uninitiated

- Let L^+ be the language you get when you add the predicate 'is true' to L
- When explaining to the speaker of L how to speak L^+ , we say:
 - You can assert 's is true' when and only when you can assert s
 - You can assert 's is false' when and only when you can deny s

Explaining Truth to the Uninitiated

- The speaker already knew when to assert or deny claims about the world, like 'Snow is white' and 'Grass is indigo'
- So our explanation of 'is true' will now allow the speaker to understand claims like:
 - 'Snow is white' is true
 - 'Grass is indigo' is false

What about Talk about Truth?

- What about sentences like the following?
 - (1) ' "Snow is white" is true' is true
- Well, we told the speaker that you can assert 's is true' whenever you can assert s
- So the speaker knows that they can assert (1) just when they can assert
 - (2) 'Snow is white' is true
- And they know that they can assert (2) just when they can assert this sentence from L , which they already understand:
 - (3) Snow is white

Groundedness

(1) "'Snow is white' is true' is true



Assert when you
are willing to assert

(2) 'Snow is white' is true



Assert when you
are willing to assert

(3) Snow is white

- Following Kripke, we can say that (1) is **grounded** in (3)

What about the Liar?

- Now suppose that the speaker tries to figure out when they can assert λ :

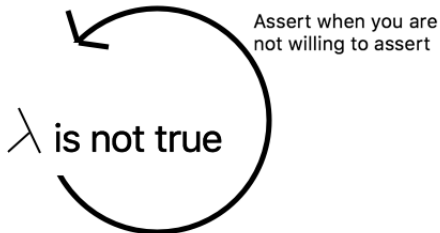
(λ) λ is not true

- They know that they can assert ' s is true' just when they can assert s , so they know that they can assert ' λ is not true' just when they cannot assert λ

What about the Liar?

- But $\lambda = \text{'}\lambda \text{ is not true'}$!
- So all they can say is that they should assert λ when they cannot assert λ
- This is obviously a contradictory instruction, but more importantly for our purposes, it is a circular one
 - In the explanation of when to assert λ , we assume that we already know when not to assert λ !

Ungroundedness



- Following Kripke, we can say that λ is **ungrounded**

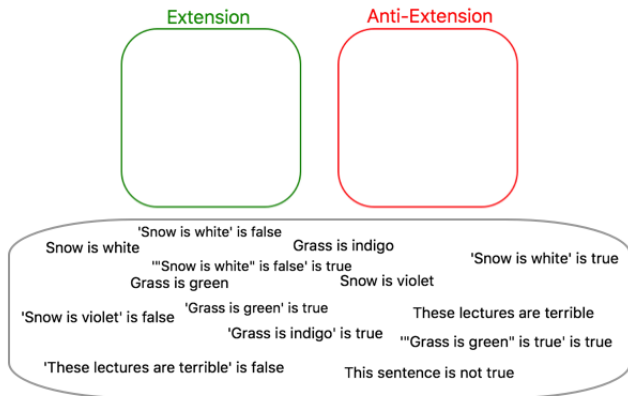
Grounded versus Ungrounded

- Kripke's idea was that claims about truth should be grounded in claims that do not deal with truth (i.e. sentences in L)
- Grounded claims are true or false, but ungrounded claims are neither
- So the Liar sentence, λ is a truth-value gap: it is neither true nor false
- To make this idea precise, Kripke had to introduce a formal theory of groundedness

A Theory of Grounding

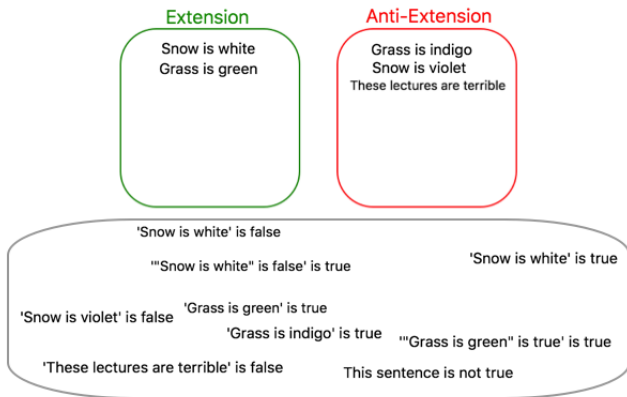
- We are used to thinking of predicates as having extensions
 - ' F ' is true of x iff x is in the extension of ' F '
 - ' F ' is false of x iff x is not in the extension of ' F '
- Now we need to modify that idea a bit:
 - 'is true' comes with an extension, E , and an **anti-extension**, A
 - A sentence is true iff it is in E
 - A sentence is false iff it is in A
 - A sentence is neither true nor false iff it is neither in E nor A

A Theory of Grounding



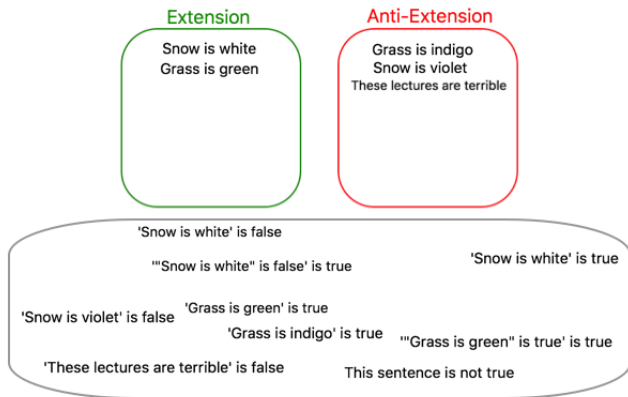
- Consider **all** of the sentences of L^+ . (This picture is a bit misleading: there are infinitely many of these sentences!)

A Theory of Grounding



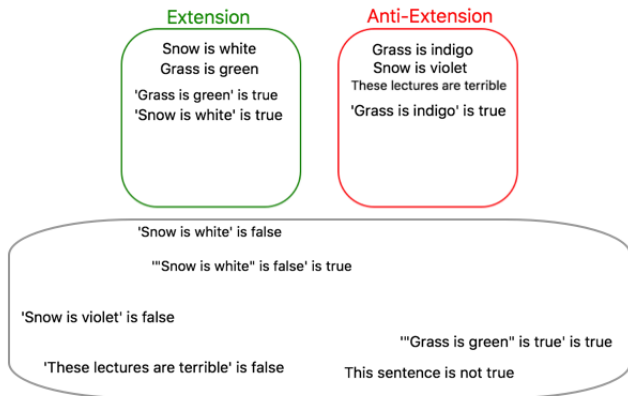
- Put all of the sentences from L that should be asserted into the extension, and all the ones that shouldn't be asserted into the anti-extension

A Theory of Grounding



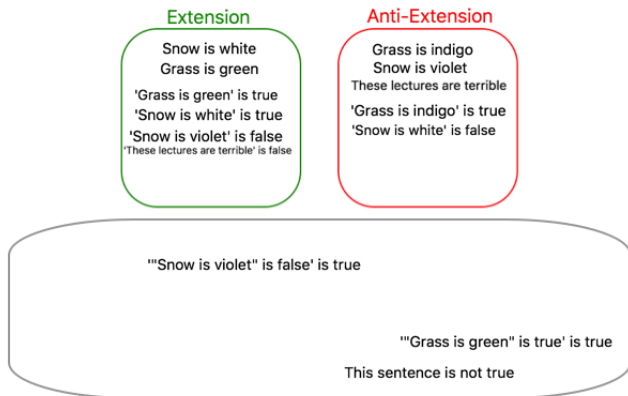
- Now we can add some sentences that mention truth into the extension and anti-extension

A Theory of Grounding



- Put 's is true' in the extension iff s is in the extension
- Put 's is true' in the anti-extension iff s is in the anti-extension

A Theory of Grounding



- Put 's is false' in the extension iff s is in the anti-extension
- Put 's is false' in the anti-extension iff s is in the extension

A Theory of Grounding

Extension

Snow is white
Grass is green

'Grass is green' is true
'Snow is white' is true
'Snow is violet' is false
'These lectures are terrible' is false
'"Grass is green" is true' is true

Anti-Extension

Grass is indigo
Snow is violet

These lectures are terrible
'Grass is indigo' is true
'Snow is white' is false
'"Snow is white" is false' is true

This sentence is not true

- Now following exactly the same rules, we can add some more sentences into the extension and anti-extension

A Theory of Grounding

- Kripke proved that if we carry this process on long enough, we will eventually find that we are not adding any more sentences to E or A
 - This is called a **fixed-point**
- We have to carry the process on a **really** long time: *infinitely* long!
 - Mind boggling, but modern mathematics allows us to talk sensibly about extending this process past infinity (“transfinitely”)

A Theory of Grounding

- Kripke's idea is that a sentence is true iff it is in E at the fixed-point, and false iff it is in A at the fixed-point
- Crucially, Kripke also proved that λ is in neither E nor A at the fixed-point
- So λ is neither true nor false!

Solving the Liar Paradox

- What about the following instance of (T)?
 - (1) ' λ is not true' is true iff λ is not true
- It depends on how we understand the biconditional, 'iff' in this context
- On Kripke's view, ' A iff B ' is neither true nor false whenever A or B is neither true nor false
- So on this view, (1) is neither true nor false
- The Liar Paradox is thus premise-flawed in the sense that one of its premises is not true (although it also is not false)

The Liar Paradox

Kripke's Solution: Truth-Value Gaps

An Objection to Kripke's Solution

The Strengthened Liar is not True!

- We first introduced the idea of truth-value gaps right at the beginning of Lecture 8
- We raised a problem:
 - If λ is neither true nor false, then it is not true
 - But λ says of itself that it is not true!
 - So if λ is neither true nor false, then it is true!
- How does Kripke get around this problem?

Negation in Kripke's System

- Kripke is forced to say that the negation in L^+ , 'not', works like this:

A	not- A
T	F
u	u
F	T

- From this truth-table for negation and Kripke's definition of truth, it follows that it is impossible to truly say that λ is neither true nor false
- If we try, then we will just end up saying another thing which is neither true nor false!

A Different Kind of Negation

- Isn't that a disaster for Kripke? His whole solution to the Liar Paradox is to say that λ is neither true nor false!
- Kripke's only way out of this mess is to insist that the notion of truth he has used to describe L^+ is **not** the same notion of truth as the one that appears *within* L^+
- We have now returned to something like Tarski's hierarchy!
- The hierarchy is less restrictive — we can now say a lot about truth without having to ascend any levels — but it is still there

The Ghost of Tarski's Hierarchy

Liar sentences are not true in the object language, in the sense that the inductive process never makes them true; but we are precluded from saying this in the object language by our interpretation of negation and the truth predicate [...] The necessity to ascend to a metalanguage may be one of the weaknesses of the present theory. The ghost of the Tarski hierarchy is still with us.

(Kripke 1975, p. 714)

References

- Kripke, S (1975) 'Outline of a Theory of Truth', *Journal of Philosophy* 72: 690–715
- Tarski, A 'The concept of truth in formalized languages' in Corcoran (ed) Woodgar (trans) *Logic, Semantics, Metamathematics* (Indianapolis, IN: Hackett, 1983) pp. 152–278