Rationality, Morality and Economics

Topic 1, Lecture 1

Decisions Under Ignorance

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Decisions Under Ignorance

What is Decision Theory?

- In this module, we will explore **decision theory** (and related topics)
- Decision theory is the study of *rational decision making*
- Decision theorists propose and criticise various decision rules, rules which tell us what option to choose in a given decision problem

What is Decision Theory Good For?

- Decision theory is an important part of modern economics
- Decision theory also has important political applications
  - Later in the term you will look at Arrow's Impossibility Theorem, which very roughly tells us that there is no ideal system for making social decisions
  - Every system is bound to violate at least one constraint on good social decision making
- Decision theory has lots of interesting philosophical implications

Decision Theory and Rationality

- Decision theory is the study of **rational** decision making
- Philosophers have been interested in the nature of rationality since philosophy began
- Questions about rationality crop up all over philosophy
  - Donald Davidson famously argued that all language users must be minimally rational, otherwise you cannot interpret them as actually *using* a *language*
- Studying decision theory is one way of studying the crucial philosophical concept of *rationality*, and that will be our focus in Topic 2

Decision Theory and Ethics

- Decision theory also has important applications in **ethics**
- Harsanyi presented two famous arguments for utilitarianism
  - **The Aggregation Theorem**, which you will look at later in the term
  - **The Impartial Observer Theorem**, which we will look at in the next lecture
- John Rawls also used decision theory while arguing for his distinctive conception of justice, and we will also look at that in the next lecture

Decision Theory and Causation

- Decision theory also has important consequences for philosophical discussions of **causation**
- We talk about cause-and-effect all the time in everyday life, but there are good philosophical questions about what causation actually is
- Some philosophers, like Bertrand Russell, have gone so far as to dismiss causation as a useless superstition
  - Advanced scientific theories never appear to mention causation
- However, some philosophers have argued that causation has a crucial role to play in decision theory
- We will ask if they are right in Topic 3

Decisions Under Ignorance

Topics and Lectures

- This module is divided into six **topics**
  - I will be leading the first three topics, and John Bone will be leading the last three
- Each topic is made up of two **lectures**
  - Tuesday 17:00–18:00, and Thursday 09:00–10:00
- **There will be no lectures on Weeks 5, 9 or 10**
  - You should use this time off from lectures to revise the material we have covered, and to delve more deeply into the reading

Seminars

- There will be a seminar for each topic
  - Wednesday 09:00–10:30, or Friday 09:00–10:30
- The seminar of a topic will take place in the week **after** the two lectures for that topic
  - For example: the lectures for Topic 1 are this week (Week 2), and the seminars for Topic 1 will take place next week (Week 3)
- **There will be no seminars on Weeks 2, 6 or 10**
  - Use this time off from the seminars to revise the material we have covered, and to delve more deeply into the reading

Preparing for the Seminars

- Specific readings have been set for each seminar
- It is **essential** that you complete these readings **before** the seminar
- It is also essential that you answer any study questions that have been set, and bring those answers to the seminars **in writing**
- Finally, it is also essential that you bring at least one **question** to each seminar, **in writing**

Readings

- The textbook for this module is Martin Peterson's *An Introduction to Decision Theory*, 2nd edition
  - There are a number of copies of this book in the library
  - We also have e-access to the first edition, but be warned: the first edition contained a great many errors
- You can also find a full reading list for the module on the VLE page

Formative Assessment

- 1,200 word essay on one topic we have covered
- You can find the essay question for each topic on its VLE page
- If you choose to write an essay on topics 1, 2 or 3, then you can submit it on the VLE site between 9am Wednesday 13th of February and 5pm Friday 15th of February
- If you choose to write an essay on topics 4, 5 or 6, then ou can submit it on the VLE site between 9am Wednesday 13th of March and 5pm Friday 15th of March

Summative Assessment

- 3 hour exam in the Summer Term
- There will be 6 questions, one on each topic, from which you choose 3 to answer
- Please see the VLE site for more details, where you will be able to find past exam papers

Decisions Under Ignorance

A Decision Matrix

Should you go to see *The Favourite*?

	Lives up to hype	Overrated
Go to see it	10	1
Stay home	1	5

A Decision Matrix

Should you go to see *The Favourite*?

	Lives up to hype	Overrated
Go to see it	10	1
Stay home	1	5

- The **acts** are the things you can choose to do in the decision problem
- It is standardly assumed that the **acts** are **mutually exhaustive** — you have to choose one of them — and **mutually exclusive** — you can only choose to do one of them

A Decision Matrix

Should you go to see *The Favourite*?

	Lives up to hype	Overrated
Go to see it	10	1
Stay home	1	5

- The **states of nature** are ways that things might turn out, which you don't control
- It is standardly assumed that the **states** are **mutually exhaustive** — one of them must obtain — and **mutually exclusive** — only one of them can obtain

A Decision Matrix

Should you go to see *The Favourite*?

	Lives up to hype	Overrated
Go to see it	10	1
Stay home	1	5

- An **outcome** is what you get when you combine an act with a state
- Agents have preferences over **outcomes**, which we have represented numerically — **outcomes** with higher numbers (or *utilities*) are preferred to **outcomes** with lower numbers

Two Types of Decision Problem

- **Decisions under risk**
  - You know the possible outcomes of each act, and you can assign subjective probabilities to those outcomes

- **Decisions under ignorance**
  - You know the possible outcomes outcomes of each act, but you cannot assign subjective probabilities to those outcomes

An Example

- Billy has a heart condition. A surgeon offers Billy a heart transplant, using her own special method. Should Billy have the heart transplant?

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- Whether this is a decision under risk or ignorance depends on what Billy knows about the surgeon's special method

An Example

- Billy has a heart condition. A surgeon offers Billy a heart transplant, using her own special method. Should Billy have the heart transplant?

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- If the surgeon has **never** tried this method before, Billy may not be able to assign probabilities to the outcomes

An Example

- Billy has a heart condition. A surgeon offers Billy a heart transplant, using her own special method. Should Billy have the heart transplant?

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- But if the surgeon has used this method many times, Billy can look at her track record to help him assign probabilities

An Example

- Billy has a heart condition. A surgeon offers Billy a heart transplant, using her own special method. Should Billy have the heart transplant?

	Method works (0.95)	Method doesn't work (0.05)
Transplant	20	0
No transplant	5	5

- But if the surgeon has used this method many times, Billy can look at her track record to help him assign probabilities

The Principle of Maximising Expected Utility

- As John explained in his online video lecture, the standard principle for decision making is the **Principle of Maximising Expected Utility**
  - You should choose the act with the greatest expected utility
  - $EU(a) = \sum_{i=1}^{n}[P(s_i) \times U(a \wedge s_i)]$

	Method works (0.95)	Method doesn't work (0.05)
Transplant	20	0
No transplant	5	5

- EU(Transplant) $= (0.95 \times 20) + (0.05 \times 0) = 19$ ✓
- EU(No transplant) $= (0.95 \times 5) + (0.05 \times 5) = 5$ ×

The Principle of Maximising Expected Utility

- As John explained in his online video lecture, the standard principle for decision making is the **Principle of Maximising Expected Utility**
  - You should choose the act with the greatest expected utility
  - $EU(a) = \sum_{i=1}^{n} [P(s_i) \times U(a \wedge s_i)]$

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- But we cannot apply the Principle of Maximising Expected Utility in cases of ignorance — we don't have the probabilities we need to calculate expected utilities!

Coping with Ignorance

- In the remainder of this lecture, we will look at three decision principles that we might try in cases of ignorance
- Peterson discusses some more decision principles in the textbook, but I am focussing on three of the most important
- We will return to decisions under risk next week, when we will ask what justifies the Principle of Maximising Expected Utility

Decisions Under Ignorance

The Maximin Rule

- One strategy for dealing with decisions under ignorance is to focus on the worst case scenario
- If we choose an act with the the best worst case outcome, then we know things can't go too badly for us
- This line of thought leads us to the **Maximin Rule**:
  - $a_i \succeq a_j$ if and only if $\min(a_i) \geq \min(a_j)$

    $a_i \succeq a_j$ means that performing act a_i is at least as rational as performing act a_j

    $\min(a_i)$ is the utility of the worst outcome obtainable by performing act a_i, and likewise for $\min(a_j)$

Applying Maximin

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- Maximin recommends not having the transplant
  - The worst outcome obtainable by having the transplant is 0
  - The worst outcome obtainable by not having the transplant is 5
- As this example illustrates, Maximin is a conservative rule

The Strong Dominance Principle

- Maximin is often criticised for contradicting the **Strong Dominance Principle**
- The idea behind Strong Dominance is that you should prefer a_i to a_j if performing a_i cannot produce a worse outcome than performing a_j, and might actually produce a better one
- Here is a formalisation of **Strong Dominance**:
  - $a_i \succ a_j$ if: $u(a_i, s) \geq u(a_j, s)$ for all states s, and there is some state s' such that $u(a_i, s') > u(a_j, s')$

    $u(a_i, s)$ is the utility of the outcome you get by performing act a_i in state s

    $a_i \succ a_j$ means that it is more rational to perform a_i than it is to perform a_j

Maximin versus Strong Dominance

- Consider Billy again, who is wondering whether to get a heart transplant
- Imagine that Billy is offered another option to choose from: if he wants, he can ask for the surgeon's **gold package**
- A successful gold package surgery will add an *extra* five years to Billy's life

	Method works	Method doesn't work
Standard transplant	20	0
Gold package	25	0
No transplant	5	5

Maximin versus Strong Dominance

	Method works	Method doesn't work
Standard transplant	20	0
Gold package	25	0
No transplant	5	5

- Intuitively, getting a gold package transplant is more rational than getting a standard transplant
- That's also the verdict of Strong Dominance
- But Maximin tells us to be indifferent between getting a gold package transplant or a standard one, because their worst case outcomes are the same

Maximin versus Strong Dominance

- Many think that Strong Dominance is a fundamental constraint on rationality, and so insist that we must reject Maximin
- We will talk more about Strong Dominance in Topic 3, and we will see that things are a little bit more complicated than that
- However, some version of Strong Dominance seems right, especially when we are dealing with decisions under ignorance
- Fortunately, there is a way of tweaking Maximin to avoid the conflict with Strong Dominance

The Leximin Rule

- The **Leximin Rule** is just like the Maximin rule, except it breaks ties
- If the worst outcome of a_i is better than the worst outcome of a_j, then Leximin tells us that we should prefer a_i to a_j
- But if the worst outcome of a_i is **no better or worse** than the worst outcome of a_j, Leximin tells us to compare the second-to-worst outcomes
- And if the second-to-worst outcome of a_i is also no better or worse than the second-to-worst outcome of a_j, then Leximin tells us to consider the second-to-worst outcomes...

The Leximin Rule

- Here is a formalisation of the **Leximin Rule**:
  - $a_i \succ a_j$ if and only if there is some positive integer n such that $\min^n(a_i) > \min^n(a_j)$, and $\min^m(a_i) = \min^m(a_j)$ for all $m < n$

    $\min^n(a_i)$ is the n-th worst outcome obtainable by performing a_i

- Leximin is compatible with Strong Dominance

Leximin and Strong Dominance

	Method works	Method doesn't work
Standard transplant	20	0
Gold package	25	0
No transplant	5	5

- The worst outcome of getting a gold package transplant is the same as the worst outcome of getting a standard transplant
- But the second-to-worst outcome of getting a gold package transplant is 25, whereas the second-to-worst outcome of getting a standard transplant is 20
- So Leximin tells us to prefer the gold package transplant to the standard transplant, just like Strong Dominance

Decisions Under Ignorance

From Ignorance to Risk

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- Suppose that Billy has absolutely no reason to think that it is more likely that the surgeon's method works than that it doesn't, and *vice versa*
- In this situation, we might suggest that Billy should just assign these two states the same probability

From Ignorance to Risk

	Method works (0.5)	Method doesn't work (0.5)
Transplant	20	0
No transplant	5	5

- Now we can use the Principle of Maximising Expected Utility
- EU(Transplant) = 10, whereas EU(No transplant) = 5
- So Billy should get the transplant!

The Laplace Rule

- This leads us to the final rule we will consider
- It is sometimes known as the *Principle of Insufficient Reason*, but we will call it the **Laplace Rule**, after the mathematician Pierre-Simon Laplace
- This rule deals with decisions under ignorance by turning them into decisions under risk

Pierre-Simon Laplace

The Laplace Rule

- If we have no reason to think that any one state is more or less probable than any other state, then we should assign all the same probability to every state
- We should then use this probability assignment to apply the Principle of Maximising Expected Utility
- We can formalise the **Laplace Rule** as follows:
  - $a_i \succ a_j$ if and only if $\sum_{x=1}^{n} \frac{1}{n} u(a_i, s_x) > \sum_{x=1}^{n} \frac{1}{n} u(a_j, s_x)$
- **Important:** The Laplace Rule is only meant to be applied when we have no reason to think that one state is more or less probable than another

Ordinal versus Interval Utility Scales

- The Laplace Rule is importantly different from both Maximin and Leximin

- Maximin and Leximin only require *ordinal* utility scales

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- All that the numbers tell us is how Billy would rank the possible outcomes
  - Getting the transplant and the method working is the best, getting the transplant but the method not working is the worst, and not getting the transplant in either state is second worst

Ordinal versus Interval Utility Scales

- But the Laplace Rule requires an *interval* utility scale
- In an interval scale, the differences between the utilities of two outcomes tells us about how much one of these outcomes is preferred to the other

	Method works	Method doesn't work
Transplant	20	0
No transplant	5	5

- If we are using an interval scale, then this table tells us that the difference between a successful transplant and an unsuccessful one is four times greater than the difference between an unsuccessful transplant and no transplant at all

Ordinal versus Interval Utility Scales

- A utility function is a function from outcomes to real numbers, where the real numbers represent the utilities of the outcomes
- If function f represents your utilities on an ordinal scale, then so does g, if and only if:

  $g(x) \geq g(y)$ if and only if $f(x) \geq f(y)$

Ordinal versus Interval Utility Scales

- A utility function is a function from outcomes to real numbers, where the real numbers represent the utilities of the outcomes

- If function f represents your utilities on an ordinal scale, then so does g, if and only if:

  g is a positive montone transformation of f

Ordinal versus Interval Utility Scales

- A utility function is a function from outcomes to real numbers, where the real numbers represent the utilities of the outcomes
- If function f represents your utilities on an ordinal scale, then so does g, if and only if:

  g is a positive montone transformation of f

- If function f represents your utilities on an interval scale, then so does g, if and only if:

  $g(x) = m \times f(x) + c$, where m is positive

Ordinal versus Interval Utility Scales

- A utility function is a function from outcomes to real numbers, where the real numbers represent the utilities of the outcomes
- If function f represents your utilities on an ordinal scale, then so does g, if and only if:

  g is a positive montone transformation of f

- If function f represents your utilities on an interval scale, then so does g, if and only if:

  g is a positive affine transformation of f

Laplace with an Ordinal Scale

- It is fairly easy to see that the Laplace Rule needs more than an ordinal utility scale

	Method works (0.5)	Method doesn't work (0.5)
Transplant	20	0
No transplant	5	5

- EU(Transplant) $= 10$ and EU(No transplant) $= 5$, so the Laplace Rule tells us that Billy should get the transplant

Laplace with an Ordinal Scale

- But now let's change the decision problem by applying the following monotone transformation to the utility function:

	Method works (0.5)	Method doesn't work (0.5)
Transplant	6	0
No transplant	5	5

- EU(Transplant) = 3 and EU(No transplant) = 5, so the Laplace Rule now tells us that Billy should not get the transplant

Laplace with an Interval Scale

- It is also easy to see that the Laplace Rule doesn't need anything more than an interval utility scale
- Simple arithmetic tells us that:
  - $\sum_{x=1}^{n} \frac{1}{n} u(a_i, s_x) > \sum_{x=1}^{n} \frac{1}{n} u(a_j, s_x)$ if and only if $\sum_{x=1}^{n} m \times \frac{1}{n} u(a_i, s_x) + c > \sum_{x=1}^{n} m \times \frac{1}{n} u(a_j, s_x) + c$
- So no matter what positive affine transformation we apply to our utility function, the Laplace Rule will always make the same recommendations about how to act

Constructing Scales

- Since the Laplace Rule requires an interval utility scale, it requires more of our concept of utility than the maximin or leximin rules
- You could construct an ordinal utility scale simply by ranking the outcomes in terms of which you prefer more
- More needs to be done to construct an interval scale
  - See chapter 5 of Peterson's textbook for one way of doing this
- This is not a serious objection to the Laplace Rule, but it is an important point which shouldn't be neglected

An Objection to the Laplace Rule

- The Laplace Rule is very sensitive to the way that we describe the states of nature
- Suppose there are 4 different ways in which the surgeon's transplant method might fail, $F_1 \ldots F_4$
- We could represent Billy's decision problem with either of the following tables:

	W	F_1 or F_2 or F_3 or F_4
Transplant	20	0
No transplant	5	5

	W	F_1	F_2	F_3	F_4
Transplant	20	0	0	0	0
No transplant	5	5	5	5	5

An Objection to the Laplace Rule

	W	F_1 or F_2 or F_3 or F_4
Transplant	20	0
No transplant	5	5

	W	F_1	F_2	F_3	F_4
Transplant	20	0	0	0	0
No transplant	5	5	5	5	5

- If we use the first table, the Laplace Rule tells us that EU(Transplant) $= 10 >$ EU(No transplant) $= 5$
- But if we use the second table, the Laplace Rule tells us that EU(Transplant) $= 4 <$ EU(No transplant) $= 5$
- So an advocate of the Laplace Rule must say that (at least) one of these tables is a *misrepresentation*

Also an Objection to Leximin

- This seems like a serious problem for the Laplace Rule
  - How could it be that one of these table is a better representation than the other?
- However, it is also important to know that the Laplace Rule is not the only decision principle which faces this problem
- Leximin does too!

Also an Objection to Leximin

- Let's return to our first decision problem, about whether or not to see *The Favourite*

	Lives up to hype	Overrated
Go to see it	10	1
Stay home	1	5

- Presented like this, Leximin tells us to solve this decision problem by going to see the film

Also an Objection to Leximin

- But now imagine that there are two ways that *The Favourite* could be overrated: it could be less funny than people say (F_1), or less interesting (F_2)

- We can now represent the decision problem with this table:

	Lives up to hype	F_1	F_2
Go to see it	10	1	1
Stay home	1	5	5

- But now Leximin tells us not to see the film!