The concept horse with no name

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1. INTRODUCTION

Frege thought that predicates refer, but he did not think that they could co-refer with singular terms. This led him to the apparently paradoxical conclusion that although ' ξ is a horse' refers to a property (or in Frege's words, a concept), it does not refer to the property *horse*: 'the property *horse*' is a singular term and so, Frege insisted, is not eligible to co-refer with the predicate ' ξ is a horse'. In accordance with Frege's own terminology, this has become known as the concept *horse* paradox.¹

Given the delicate position it left him in, one naturally wonders why Frege thought that terms and predicates could not co-refer. Why did he think that the world would reflect the linguistic distinction between terms and predicates? It is tempting to answer this question by appealing to Frege's famous metaphor of saturation: no predicate co-refers with a term because the referents of terms are saturated and the referents of predicates are not. However, it is hard to explain what it means to say that the referents of predicates are unsaturated without simply presupposing that terms and predicates cannot co-refer. A more helpful answer is given by Wright (1998). He points out that the *Reference Principle*,

Co-referring expressions are intersubstitutable salva congruitate in all contexts,²

underlies Frege's general approach to semantics. Singular terms and predicates are not everywhere intersubstitutable *salva congruitate*. 'Shergar is a horse', for example, is wellformed, but 'Shergar the property *horse*' is not. So the Reference Principle entails that predicates do not co-refer with terms.³

While Wright's answer is an improvement on an appeal to the 'saturation' metaphor, I do not think that it traces the concept *horse* paradox all the way back to its source. In this paper I will argue that the paradox can get going without relying on the Reference

¹The *locus classicus* of this paradox is Frege's 'On concept and object' (1892), but he also discusses it elsewhere, most notably in 'Comments on sense and reference' (1891–5).

 $^{^{2}}$ In fact, this is just one half of what Wright (1998: 73) calls 'the Reference Principle'; the other is the principle that co-referring expressions are intersubstitutable *salva veritate* in extensional contexts. For my purposes, however, it will suffice to focus solely on the half of the Reference Principle presented above.

 $^{^{3}}$ The role of the Reference Principle in Frege's thinking about the concept *horse* paradox is most visible in his (1892: 189).

Principle as a premise. I will then use this conclusion to show that Hale and Wright's attempts to avoid the paradox by either rejecting or finessing the Reference Principle are unsuccessful. But before we get going, I should mention that my interests here are primarily philosophical, not historical: my aim is not to reconstruct Frege's own understanding of the concept *horse* paradox, but to uncover the philosophical roots of that paradox.

2. Preliminaries

We begin with some preliminaries. Take the sentence 'Shergar is a horse'. As is no doubt familiar, Frege divided this sentence into two parts: the singular term 'Shergar' and the predicate ' ξ is a horse'. These two parts play complementary roles in this sentence. Roughly, 'Shergar' picks out an object — Shergar — and ' ξ is a horse' says something of it — that it is a horse. (This characterisation is crude but it will do for now.) In Fregean terms, the distinction between terms and predicates is a distinction in the kinds of *sense* that expressions can have.⁴ The sense of a subsentential expression is that expression's role in determining the senses of the sentences in which it occurs, and the sense of a sentence — a *proposition* or *thought* — is that sentence's truth-condition.⁵ The different kinds of role that terms and predicates play are precisely different roles in determining the truth-conditions of the sentences in which they appear.

The predicates that I have been concerned with so far are properly described as *first-level monadic* predicates. Predicates of this type are generated by replacing one or more occurrences of a singular term in a sentence with the same gap (or variable). But there are many other kinds of predicate. First-level *polyadic* predicates are generated by replacing several terms in a sentence with different gaps. Second-level predicates are generated by replacing one or more first-level predicates with gaps; for example, the second-level predicate ' $\forall x \phi x$ ' can be extracted from ' $\forall x x$ is a horse' by replacing ' ξ is a horse' with a gap. We can continue in this way, extracting ever higher level predicates. My discussion, however, will focus on first-level monadic predicates, and so I will use the unqualified 'predicate' for, and only for, predicates of that variety. I will use ' ξ ' and ' ζ ' to mark gaps for singular terms and ' ϕ ' and ' ψ ' to mark gaps for first-level monadic predicates.

For the purposes of this paper, an *object* is anything which can be referred to with a singular term, and a *property* is anything that can be referred to with a predicate. (Frege famously used the word 'concept' rather than 'property'. I have chosen 'property' to avoid the, well, *conceptual* connotations of 'concept'.) Using these definitions, Frege's claim that terms and predicates cannot co-refer becomes the claim that no property is an object. Of course, it is by no means obvious that there really are any properties in the above sense; in other words, it is not obvious that predicates really do refer to anything.

 $^{{}^{4}}See$ for example (Frege 1923: 393).

⁵See (Frege 1893: §32) and (Dummett 1981b: 249–51). If we prefer, we could think of the sense of a sentence as its assertability-condition.

Why should we think that in saying something of an object, a predicate manages to refer to an extralinguistic correlate of its own? Nonetheless, in this paper I will take it for granted that at least some predicates refer; after all, the concept *horse* paradox is first and foremost a paradox for those who believe in predicate reference.

3. Substitution at the level of sense

In what follows, I am going to place considerable weight on the claim that terms and predicates are nowhere intersubstitutable. In short, I am going to argue that the impossibility of such substitutions prevents us from so much as expressing the thought that a property is an object. As this claim is such an important premise of my argument, I must pause on the correct way to understand it.

One thing that we might mean by this claim is that when we try to substitute a term for a predicate in a well-formed English sentence, what we end-up with is not itself a wellformed English sentence, and the same happens when we try to substitute a predicate for a term. To repeat an earlier example, when we substitute 'the property *horse*' for ' ξ is a horse' in the sentence 'Shergar is a horse', we end up with the jumble of words 'Shergar the property *horse*'.

However, my argument cannot rest on the impossibility of such straightforward grammatical substitutions in English. To begin with, as Oliver (2005: 184) pointed out when criticising the Reference Principle, 'I' and 'me' are nowhere grammatically intersubstitutable, and yet they clearly co-refer (in any given context). More generally, it is hard to see what sort of philosophical significance the failure of grammatical substitutions in English, or any particular language, can have. Certainly we could construct a language in which strings like 'Shergar the property *horse*' would count as well-formed sentences.

What is needed is a deeper sense in which terms and predicates are nowhere intersubstitutable. Fortunately, there is one to hand. Recall that the distinction between terms and predicates was characterised via their contrasting kinds of sense: roughly, terms pick out objects and predicate say things of them. So characterised, it is obvious that we can express a thought by (properly) combining an expression that has the sense of a term with an expression that has the sense of a predicate. But it seems equally obvious that we cannot express a thought by combining two expression that both have the senses of terms or two expression that both have the senses of (first-level) predicates. By the same token, we cannot express a thought by combining an expression which has the sense of a term with an expression that both have the sense of a quantifier (or more generally, of a *secondlevel* predicate): quantifiers bind argument places, and so can be meaningfully combined only with expressions that have argument places. We could put this by saying that it is impossible to substitute terms and predicates *at the level of sense*. This is how I will understand the claim that terms and predicates are nowhere intersubstitutable. Much more needs to be said about the idea that terms and predicates are nowhere intersubstitutable at the level of sense, but I will not pursue this issue too much further here. I will, however, make one salient point. The fact that terms and predicates are nowhere intersubstitutable at the level of sense has nothing to do with the parochial grammatical limitations of any given language; rather, it is due to the different kinds of sense that characterise the term/predicate distinction. So, the fact that 'I' and 'me' are not grammatically intersubstitutable in English does not entail that they are not intersubstitutable at the level of sense; in fact, I find the thought that they are not intersubstitutable at this level quite counter-intuitive.⁶ Moreover, substitution failures at the level of sense need not be detectable in the grammar of a given language at all. A language could allow us to use one and the same expression sometimes as a term and sometimes as a predicate. The point is, however, that when an expression is used in these different ways, it is being used to express different kinds of sense.⁷

4. Benno and the Misguided Metaphysician

In this section I will attempt to ignite the concept *horse* paradox without presupposing the Reference Principle. I will do so by arguing that as terms and predicates are nowhere intersubstitutable (at the level of sense),⁸ we cannot make sense of identifying a property with an object. Of course, the conclusion that it is *nonsense* to say that a property is an object is not quite the same as the conclusion that it is *false* to do so, but it is enough to generate the air of paradox: although ' ξ is a horse' refers to a property, it is nonsensical to say that ' ξ is a horse' refers to the property *horse*. What is more, when Frege was careful he was clear that the proper formulation of his position was that it is impossible to suppose that some property is an object.⁹ In §§5–6 I will consider objections to this argument.

Before going any further, I must lay down a ground rule. I will not consider the suggestion that we can make sense of identifying properties with objects by appealing to thoughts that cannot, even in principle, be linguistically expressed. This rule could be justified either theoretically or dialectically. Theoretically, we could argue that all thoughts can be expressed linguistically. This would not amount to arguing that no one can grasp a thought which they themselves cannot express linguistically; rather, the point would be that there is no thought which in principle could not be linguistically expressed by anyone. Dialectically, we could point out that whatever else we are doing when we do philosophy, we are trying to express our thoughts linguistically; we cannot appeal to

⁶Indeed, it seems plausible to suggest that 'I' and 'me' have exactly the same sense; the difference between these words would then be a product of the particular grammar of English. If that is right, then 'I' and 'me' are trivially intersubstitutable at the level of sense.

 $^{^{7}}$ See also (Frege 1892: 189).

⁸From now on I will usually leave this qualification tacit.

⁹See for example (Frege 1892: 189; 1891–5: 175).

the ineffable in philosophy because philosophy is an exercise in effing. But whichever justification we prefer, this rule will be observed throughout this paper.

Suppose we come across a man, Benno, who apparently believes that a property is identical to an object. For the sake of definiteness, let the property be the one referred to by the predicate ' ξ is a horse' and the object be the one referred to by the term 'the property *horse*'. (I assume that we can make sense of the general claim that some property is an object only if we can make sense of an instance of it; the particular instance that I have chosen is inconsequential.) As we are ignoring appeals to ineffable thoughts, our task is to see if we can, in principle, express Benno's putative thought in language. I should warn, though, that we will go about this task in the only way that we can: by jumping into Benno's way of speaking with both feet. I will briefly return to this issue in §8.

So, how is Benno to articulate his apparent thought? Well, he is immediately faced with a difficulty. Ordinarily, if we want to express an identity, we just write two terms into the gaps in the identity predicate, ' $\xi = \zeta$ '. But Benno cannot simply plug ' ξ is a horse' into the gap in ' ξ = the property *horse*': that gap is a gap for singular terms, and we cannot substitute a predicate for a term. At first this problem may not sound too serious. All that Benno needs to do is find a surrogate term for ' ξ is a horse', a term which for present purposes can do duty for ' ξ is a horse', and then complete the predicate ' ξ = the property *horse*' with that surrogate. Let us use '*a*' schematically for whatever surrogate Benno chooses. So Benno says,

(1) a = the property horse.

But in fact, Benno gives the game away the moment that he shifts from using the predicate ' ξ is a horse' to the surrogate term '*a*'. Compare Benno's case to the following one. There is a philosopher, the Misguided Metaphysician, who believes that Julius Caesar is the number 3. However, rather than expressing her thought by saying 'Julius Caesar = 3', the Misguided Metaphysician tries to do it by saying,

(2)
$$2+1=3$$
.

It is obvious (I hope!) that the Misguided Metaphysician does not express her thought when she says (2). This is made manifest by the fact that we can accept (2) without thereby agreeing that Julius Caesar is the number 3. What is more, this would still be true even if 2 + 1 in fact referred to Julius Caesar. Of course, if 2 + 1 did refer to Julius Caesar then (2) would say (de re) of Julius Caesar that he is the number 3, but it would not say *that* Julius Caesar is the number 3. The problem with trying to say that Julius Caesar is the number 3 by saying (2) is that whether or not 2 + 1 refers to Julius Caesar, it obviously does not *present its referent as Julius Caesar*. The Misguided Metaphysician's failure to express her thought, then, is due to the fact that even if she has picked out Julius Caesar with 2 + 1, she has not picked him out in the way that she must if she is to express that thought.

The attempt to express Benno's thought with (1) is importantly analogous to the attempt to express the Misguided Metaphysician's thought with (2). Both Benno and the Misguided Metaphysician think that one thing is presented in two radically different ways. Let's say that singular terms present their referents nominally, and that predicates present their referents *predicatively*. In this terminology, Benno apparently thinks that one thing is presented both nominally and predicatively, just as the Misguided Metaphysician thinks that one thing is presented both as a number and as Julius Caesar. The problem for the Misguided Metaphysician was that when she asserted (2), she did not express an identity between something presented as a number and something presented as Julius Caesar; she presented both of the arguments to her identity as numbers. Similarly, when Benno asserted (1) he did not express an identity between something presented nominally and something presented predicatively; he presented both of the arguments to his identity nominally. He has, therefore, no more managed to express his thought with (1) than the Misguided Metaphysician managed to express her thought with (2). This fact is made manifest by the fact that we can accept (1) without thereby agreeing to Benno's thought: (1) merely expresses an identity between two things presented nominally. Moreover, in presenting this argument I am not just assuming from the outset that 'a' does not refer to the property referred to by ' ξ is a horse'. The difficulty for Benno is that even if he has managed to refer to a property with 'a', he has not presented that property in the way that he must to express his thought; to do that he must refer to the property with a predicate.

This last point is worth underlining. The difficulty that I am raising for Benno is not initially aimed at the level of reference. I am not assuming that because Benno used the singular term 'a', he did not refer to a property. That assumption would be straightforwardly question begging in the current context. Rather, the objection is aimed initially at the level of sense. When Benno asserted (1), he did not express the thought that he was after, just as the Misguided Metaphysician did not express her thought with (2). Nonetheless this sense-level failure has reference-level consequences: unless we can find a way of expressing Benno's thought, we cannot suppose that it might actually be true. But crucially, the argument did not employ the premise that singular terms cannot refer to properties.

5. Two tricky surrogate terms

It is tempting to object that Benno could dodge the problem that I posed above by simply choosing his surrogate term wisely. In particular, it seems as though Benno could express his thought by using either 'the referent of " ξ is a horse" ' or 'the property *horse*' as his

surrogate for ' ξ is a horse'. In this section I will show what is wrong with this objection.

We start with 'the referent of " ξ is a horse" '. So, imagine that Benno says,

(3) The referent of ' ξ is a horse' = the property horse.

Why would we think that (3) expresses Benno's thought? Well, properties are those things which predicates can refer to. In particular then, the referent of ' ξ is a horse' is a property. And so, (3) says that a particular property is identical to a particular object.

The cogency of the above line of thought turns on how we should understand 'refers' in 'a predicate refers to a property'. If we should read it as ' ξ refers to ζ ', as we do in '"Julius Caesar" refers to Julius Caesar', then it is wholly compelling. But in fact, the discussion of §4 already suffices to show that this is not how to read 'refers' in this context. To see this we must temporarily put predicates to one side and investigate the relation between singular reference and disquotation. Take the following disquotation sentence,

(4) The referent of 'Julius Caesar' = Julius Caesar.

Everyone accepts that (4) is obviously true.¹⁰ (What people argue over is whether there is anything to the notion of singular reference that is not captured by disquotation principles.) But *why* is (4) so obviously true? We can answer this question by reminding ourselves of what we are trying to do when we use the words 'the referent of "Julius Caesar". Our ambition is to talk about the semantics of our language; we want to stop just using 'Julius Caesar' to refer to something, and start talking explicitly about what the term 'Julius Caesar' refers to. If we are to fulfil this ambition, it is obviously necessary that what we refer to with the words 'the referent of "Julius Caesar" is be the same thing that we refer to when we actually use 'Julius Caesar'. More simply put, 'the referent of "Julius Caesar" and 'Julius Caesar' co-refer. Now, given the standard rules for deciding the truth-value of an identity sentence, (4) is true if and only if 'the referent of "Julius Caesar" is one to somehow misfire, (4) must be true.

To be clear, although (4) is true if and only if 'the referent of "Julius Caesar" ' and 'Julius Caesar' co-refer, (4) does not *say* that those terms co-refer. If we wanted to say that we would have to use a sentence like,

(5) The referent of 'the referent of "Julius Caesar" '= the referent of 'Julius Caesar'.

But it is important to note that there is a sense in which (4) is more fundamental than (5). To see that, try supposing that (4) is false. As I said above, on this supposition our use of the words 'the referent of "Julius Caesar" 'misfires: when we use 'the referent of

 $^{^{10}}$ At least, (4) is obviously true when our metalangauge is (an extension of) our object-language. In this paper I will ignore the case where the metalanguage is not (an extension of) the object-language; however, I believe that we could extend my argument to cover this case if we helped ourselves to an appropriate notion of translation.

"Julius Caesar", we do not manage to refer to the referent of 'Julius Caesar'. But if our use of 'the referent of "Julius Caesar" is misfires in this way, then so does our use of (5). When we assert (5) we attempt to say something about the referent of 'Julius Caesar', and we do so by using the words 'the referent of "Julius Caesar" ; our attempt is successful, then, only if 'the referent of "Julius Caesar" is successfully picks out the referent of 'Julius Caesar'. So, (5) says what it is meant to say only if (4) is true. We can put the difference between (4) and (5) in the following way. When we assert (4) we link our use of 'the referent of "Julius Caesar" is only if that link is in place that (5) says that the referent of "Julius Caesar" is the referent of 'Julius Caesar'.

Let us now return to predicates. If we are to count the predicate ' ξ is a horse' as a referring expression, then it is essential that our attempts to talk explicitly about what ' ξ is a horse' refers to do not misfire in the way we just imagined our use of 'the referent of "Julius Caesar" 'misfiring; in other words, it is essential that when we move from using ' ξ is a horse' to trying to talk explicitly about what ' ξ is a horse' refers to, we do not inadvertently start referring to a different thing. We have already seen that our use of 'the referent of "Julius Caesar" 'works as it should only if (4) is true. So what we need now is an analogue of (4) for ' ξ is a horse'.

Given our ban on the ineffable, there must be some way of linguistically expressing this analogue of (4) for ' ξ is a horse'. But if we read 'refers' for predicates as ' ξ refers to ζ ' and then tried to produce that analogue, we would face a rerun of the general problem discussed in §4. Expressing the required analogue would be a matter of expressing an identity between something presented predicatively by ' ξ is a horse' and something presented nominally by 'the referent of " ξ is a horse". But it is impossible to substitute ' ξ is a horse' into the gap in 'The referent of " ξ is a horse" = ξ '. The best we could do, then, would be to choose some surrogate singular term and plug that into the gap instead. And once again, no choice of surrogate would allow us to express the thought that we are after. As before, this is a problem which initially occurs at the level of sense, not reference. If we plugged a surrogate term into 'The referent of " ξ is a horse" = ξ ', we would express an identity between two things presented nominally; but to repeat, an analogue of (4) would express an identity between something presented nominally by ' ξ is a horse'.

At this point Benno may offer the following response. The argument that I have offered against reading 'refers' for predicates as ' ξ refers to ζ ' is a rerun of the argument in §4. But in that case, can't Benno dodge this version of the problem by simply choosing his surrogate term wisely again? In particular, it is tempting to think that Benno could express the desired analogue of (4) by ascending once more and saying,

(6) The referent of 'the referent of " ξ is a horse" '= the referent of ' ξ is a horse'.

Unfortunately for Benno, (6) does not express the thought about predicate reference to which I have been gesturing. We can see that (6) falls short by reminding ourselves that we cannot replace (4), the disquotation sentence for 'Julius Caesar', with (5). When we assert (5) we attempt to say that 'the referent of "Julius Caesar' and 'Julius Caesar' co-refer. But as we saw, this attempt is successful only if (4) is true. Similarly, when Benno asserts (6) he attempts to say that 'the referent of " ξ is a horse' and ' ξ is a horse' co-refer. But again, the success of this attempt depends on the truth of a further principle; this principle would actually link our use of 'the referent of " ξ is a horse' ' to our use of ' ξ is a horse'. The problem for Benno is that he cannot put such a principle into words.

We should not, then, read 'refers' in 'a predicate refers to a property' as ' ξ refers to ζ '. But how should we read it? Well, the problem we have just been discussing is a consequence of the fact that when we read 'refers' in this context as ' ξ refers to ζ '. we specify what a predicate refers to with a singular term, not a predicate. It would not come up, then, if we read 'refers' for predicates as ' ξ refers to ϕ ',¹¹ where the gap marked by ' ϕ ' is a gap for predicates; if we do, we will use a predicate to specify what a predicate refers to, in the disquotational case we will use the very predicate whose referent is being specified. Of course, things cannot be left here. How are we to understand this strange new predicate ' ξ refers to ϕ '? Indeed, it is not even grammatical in English to write '" ξ is a horse" refers to is a horse'.¹² Dummett (1981a: 217) has attempted to address this sort of concern by providing an acceptable (logicians') English rendering of ' ξ refers to ϕ ': he suggested ' $\forall x(x \text{ is what } \xi \text{ refers to } \leftrightarrow \phi x)$ ', where the 'is' is the 'is' of predication, not identity. Now, whether this rendering, or any other, is acceptable is a matter of controversy.¹³ Nonetheless, the point remains that if we want to believe in predicate reference without generating the problem discussed above, then we must find some way of reading 'refers' in 'a predicate refers to a property' as ' ξ refers to ϕ '.¹⁴

Once we read 'refers' for predicates in this way, we cannot construct the singular term 'the referent of " ξ is a horse" 'by binding the variable in '" ξ is a horse" refers to ϕ ' with an ordinary definite description operator: such an operator binds term variables, not predicate variables. Still, there seems to be no harm in introducing a higher-level definite description operator which binds (first-level) predicate variables rather than term

 $^{^{11}}$ I am here making the standard assumption that predicates are objects. I suspect that this assumption is false, but one thing at a time!

¹²It is, however, grammatical to write ' ξ is a horse" refers to horses', and it is well known that plurals in English at least sometimes play the role of predicates. Nonetheless, it is still not clear how to understand this sentence, nor is it clear that 'horses' is playing the role of a predicate in this particular sentence.

¹³Dudman (1976: §II), Wiggins (1984: 316–7), Gaskin (1995: 164–6) and Wright (1998: 77–81) all criticise Dummett's proposal.

¹⁴In fact, this claim is a little premature, and will not be fully justified until §6.

variables; such an operator would take a second-level predicate and return a first-level predicate standing for the unique property which satisfies that second-level predicate, if there is such a unique property.¹⁵ We could then use this operator to form a higher-level definite description by binding the variable in '" ξ is a horse" refers to ϕ ', which we could write as 'the *F* such that " ξ is a horse" refers to *F*'. But crucially, 'the *F* such that " ξ is a horse" refers to *F*' would itself be a predicate, and so we could no more plug it into the gap in ' ξ = the property *horse*', which is a gap for singular terms, than we could plug ' ξ is a horse' into that gap.¹⁶ So, the line of thought presented at the beginning of this section, which seemed to show that Benno could express his thought merely by choosing 'the referent of " ξ is a horse" 'as his surrogate term for ' ξ is a horse', falls apart.

It may be useful to review the foregoing argument. Importantly, I have not simply assumed that 'refers' for predicates must be read as ' ξ refers to ϕ '. If I had then I would have begged the question. Rather, the dialectic went as follows. In §4 I presented a general argument purporting to show that we cannot make sense of identifying a property with an object. It was objected that this argument just has to be mistaken: predicates refer to properties, and so the singular term 'the referent of " ξ is a horse" ' must refer to a property. I responded to this objection by pointing out that it relied on reading 'refers' for predicates as ' ξ refers to ζ ', and that the argument of §4 *already* suffices to show that this is not how to understand 'refers' for predicates: if that was how we understood it, then we would be unable to express a thought which is to a predicate as (4) is to 'Julius Caesar'.

We come now to that other surrogate term, 'the property *horse*'. Suppose, then, that Benno tries to express his thought by saying,

(7) The property horse = the property horse.

The planet Neptune is a planet, the volcano Vesuvius is a volcano, and so isn't the property horse a property? If it is, then doesn't (7) identify a property with an object, even if only trivially? In fact, we are now in a position to see that the analogy between the terms 'the planet Neptune' and 'the volcano Vesuvius' on the one hand and the term 'the property horse' on the other gives out at a crucial point. ' ξ is a planet' and ' ξ is a volcano' are first-level predicates, which is why we can unproblematically plug the terms 'the planet Neptune' and 'the volcano Vesuvius' into them. But if we read 'refers' in 'a predicate refers to a property' as ' ξ refers to ϕ ', then we must likewise read 'is a property' as a second-level predicate, ' ϕ is a property', rather than as the first-level predicate it appears to be: properties are what predicates refer to, and so the argument place in 'is a property' must match the second argument place in ' ξ refers to ϕ '. Again, it is

 $^{^{15}}$ In order to explain fully how such a higher-order definite description operator works, we would have to say something about how to talk about identity in relation to properties. This is something that I will discuss in §6.

 $^{^{16}}$ Compare (Wright 1998: 79).

controversial whether there is any acceptable way of reading 'is a property' as a secondlevel predicate. Dummett (1981a: 216) suggested that we read it as ' $\forall x (\phi x \lor \neg \phi x)$ ', but that suggestion has not been met with universal approval. The point remains, however, that once we read 'refers' for predicates as ' ξ refers to ϕ ', we have to read 'is a property' as ' ϕ is a property'. In that case, we cannot plug the singular term 'the property *horse*' into the gap in ' ϕ is a property', and so we cannot say that the property *horse* is a property in the sense Benno requires.

6. Lopsided identity

There is one last objection to my argument that I must consider. It begins by pointing out what might appear to be an intolerable problem with the position for which I have been arguing. I have been trying to show that we cannot make sense of identifying a property with an object and, as a result, that we can make no sense of referring to a property with a singular term. But if we cannot refer to properties with singular terms, then we cannot even talk about whether one property is or is not identical to *another property*: the gaps in ' $\xi = \zeta$ ' are gaps for singular terms, and so no predicates can go in them. But if we cannot talk about identity in relation to properties, in what sense are properties a kind of thing? After all, Quine was surely onto something with his famous slogan, No entity without identity.

If we have no choice but to read all talk of identity as univocally referring to a relation between objects, that relation picked out by ' $\xi = \zeta$ ', then this challenge is unanswerable. However, just as we need to find a way of distinguishing between reference-talk in relation to terms and in relation to predicates, we must find a way of distinguishing between identity-talk in relation to objects and in relation to properties. What we need is a second-level analogue of ' $\xi = \zeta$ ', that is a predicate ' $I(\phi, \psi)$ ' which is to properties as ' $\xi = \zeta$ ' is to objects. In fact, Frege (1891–5: 175–6) himself recognised the need for such an analogue of identity. He famously suggested that ' $\forall x(\phi x \leftrightarrow \psi x)$ ' would suffice, but we need not follow him in individuating properties extensionally; a natural alternative would be ' $\Box \forall x(\phi x \leftrightarrow \psi x)$ '.¹⁷ Now, if we decide to retain the word 'identity' for the relation referred to by ' $\xi = \zeta$ ', then we still cannot quite agree with Quine, but we can meet him halfway: No analogue of entity without an analogue of identity.

Recognising a second-level analogue of identity is in itself of no help to Benno: 'the property *horse*' can no more be plugged into the gap in ' $I(\phi, is a horse)$ ' than ' ξ is a horse' can be plugged into the gap in ' ξ = the property *horse*'. But now Benno might make the following suggestion. Just as we can introduce (or perhaps, recognise) a second-level analogue of identity, we can introduce (recognise) an inter-level analogue of identity,

¹⁷Hale (2013b: 142) recommends that we take ' $\Box \forall x (\phi x \leftrightarrow \psi x)$ ' as a second-level analogue of identity; however, he also thinks that we can refer to properties with singular terms and so can also use ' $\xi = \zeta$ ' in relation to properties.

which I will write as ' $\phi = \xi$ '; one candidate would be ' $\Box \forall x (\phi x \leftrightarrow x \text{ instantiates } \xi)$ '. If we had such a lopsided identity relation, we could express Benno's thought with,

(8) Is a horse \doteq the property horse.

(We could also use ' $\phi = \xi$ ' to formulate a disquotation sentence for a predicate while still reading 'refers' for predicates as ' ξ refers to ζ '.)

The question, then, is whether there can be such a lopsided analogue of identity. To answer it, we must ask ourselves what we want of this analogue. It is not enough that = be any old mapping from properties to objects; all parties in this debate are free to accept that there are such mappings, *modulo* Russell's Paradox. I am going to assume that an analogue of identity must bear certain formal similarities to identity proper, i.e. the relation referred to by ' $\xi = \zeta$ '. In particular an analogue of identity must satisfy some version or other of reflexivity, symmetry, transitivity and Leibniz's Law. Indeed, the fact that ' $\forall x (\phi x \leftrightarrow \psi x)$ ' satisfied versions of these principles in extensional contexts seems to be why Frege (1891–5: 173) thought it would suffice as a second-level analogue of identity.

Let us focus on reflexivity; if = is to serve as our lopsided analogue of *identity*, it must surely relate each property to itself (and not to anything else).¹⁸ But given our ban on ineffable thoughts, we cannot so much as entertain the thought that = is reflexive unless there is some way of expressing that thought. And how are we to do that? We cannot do it by writing ' $\forall F(F = F)$ ': the moment we tried to take an instance of 'F = F' we would end up trying to put a predicate into a gap for a singular term. More generally, as the first gap in ' $\phi = \xi$ ' is for predicates and the second is for terms, we cannot say that = is reflexive, or even that a particular property bears = to itself, unless we can *already* express identities between properties and objects. But it was in order to express just such an inter-level identity that Benno introduced ' $\phi = \xi$ ' in the first place. Benno's attempt to express his thought therefore travels in a circle.

Of course, we may doubt whether this circle is all that vicious. If Benno needs to express identities between properties and objects in order to say that = is reflexive, why can't he use ' $\phi = \xi$ ' itself to express them? As far as I can understand this response, it seems to be based on the idea that we can say that = is reflexive with something like the following sentence:

(9) $\forall F(F \doteq \text{the } x \text{ such that } (F \doteq x)).$

The thought here is that if $\phi = \xi'$ did stand for a lopsided analogue of identity, then the x such that (F = x) would be identical to F, and so (9) would say that each F bears = to itself. However, it is fairly easy to see that (9) does not really say that = is reflexive.

¹⁸I am here assuming that if *any* property were an object, then *every* property would be an object. If you do not want to make this assumption, then you should weaken the requirement that = be reflexive as follows: for any F, if F bears = to anything then it must bear = to itself. For ease of expression I will leave this complication out of the main text.

Rather, (9) is guaranteed to be true whenever ' $\phi \coloneqq \xi$ ' stands for a total functional relation from properties to objects, and in general, total functional relations need not be reflexive. To repeat a point made earlier, all parties in this debate are free to accept that there are functional relations from properties to objects. The circularity that Benno has found himself in is, then, vicious after all.

In the end, there appears to be no way of saying that = is reflexive. On the other hand, there is no difficulty in saying that our second-level analogue of identity, stood for by $(I(\phi, \psi))$, is reflexive: $(\forall F(I(F, F)))$ will do the trick. And the same goes for all of the other principles that I listed earlier: symmetry, transitivity and Leibniz's Law. The analogy between $\xi = \zeta$ and $\phi = \xi$ is, then, too weak for the latter to be regarded as a lopsided analogue of the former.

7. HALE, WRIGHT AND THE REFERENCE PRINCIPLE

In §§4–6 I have tried to show that the concept *horse* paradox can be generated without appeal to the Reference Principle:

Co-referring expressions are intersubstitutable salva congruitate in all contexts.

This marks a significant difference between my understanding of the paradox and that of Hale and Wright; they believe that it is the Reference Principle that draws us into paradox.¹⁹ Moreover, both Hale and Wright think that we cannot simply live with the paradox: in particular, they think that we cannot believe in properties and at the same time insist that singular terms cannot refer to them. As a result, they attempt to find a way around this conclusion by denying, or at least defanging, the Reference Principle. In this section I will use the the preceding discussion to show that their attempts to allow singular reference to properties are unsuccessful.

In his (1998: 76), Wright insists that the Reference Principle be respected by any decent response to the concept *horse* paradox. He therefore sets himself the task of allowing singular terms to refer to properties without violating that principle. He (pp. 84–90) does so by distinguishing between reference for singular terms, which he simply calls 'reference', and reference for predicates, which he calls 'ascription'.²⁰ Now, the Reference Principle tells us that as singular terms and predicates are not everywhere intersubstitutable *salva congruitate*, they cannot co-refer. Similarly, what we might call the Ascription Principle, according to which co-ascribing expressions are everywhere intersubstitutable *salva congruitate*, tells us that singular terms and predicates cannot co-ascribe. But neither principle, taken separately or together, entails that no singular term refers to what some

¹⁹They (2012: \S IV–VI) do ask whether there is something deeper that might motivate us to accept the Reference Principle; however, they then set about showing that there is not.

 $^{^{20}}$ In fact, Wright (1998: §VII) argues that the Reference Principle *itself* demands that we distinguish between reference and ascription. I will not discuss that argument here.

predicate ascribes.²¹

Hale (2010: 414–5) then offers his own response to the concept *horse* paradox. Rather than treating the Reference Principle as sacrosanct, he (p. 416) responds to the paradox by rejecting the principle and allowing ' ξ is a horse' to straightforwardly co-refer with 'the property *horse*'. This, Hale (p. 415) argues, has the virtue of allowing us to have a single universal reference relation for all types of expression. However, Hale is also keen to avoid giving up on the Fregean slogan 'No property is an object'. To this end, he (pp. 415–6) gives a new characterisation of objects and properties. Rather than thinking of objects as those things that we can refer to with singular terms, he asks us to think of them as those things that we can *primarily* refer to with singular terms; similarly, properties are to be those things that we can *primarily* refer to with predicates. His thought is that while singular terms can refer to properties, they do so *secondarily* or *derivatively*; the derivative nature of this singular reference to properties is meant to be revealed by the fact that our ability to use terms like 'the property *horse*' is to be explained via our ability to use predicates like ' ξ is a horse'.²²

So far things have been relatively straightforward. Unfortunately, Hale and Wright's joint paper (2012) has introduced a confusion that must be cleared up. First, they (p. 93) make a terminological change. They stop using 'the Reference Principle' as Wright did in his (1998), and use it instead for,

(10) An entity of a certain *kind* is anything which *can*, *and only can*, be referred to by an expression of a certain correlative logico-syntactic *type*.

A little while later (p. 105) they tell us that (10) is in fact the conjunction of two independent principles:

Single Relation: some *one reference relation* uniformly connects expressions of each syntactic type with the kinds of entity that provide their respective semantic values;

Type-Kind Uniqueness: syntactic types of expression correlate one-to-one with the kinds of entity among which their tokens are eligible to refer.

It is not entirely clear to me that (10) really is equivalent to the conjunction of these two principles, but we need not pause too long on this issue. Hale and Wright do all of their serious work with Single Relation and Type-Kind Uniqueness, and so we can just stipulate that the official 2012 Reference Principle is the conjunction of these principles. Wright is then characterised as rejecting Single Relation whilst retaining Type-Kind Uniqueness

 $^{^{21}}$ The idea that predicates ascribe rather than refer to properties is further developed by Liebesman in his (forthcoming).

²²Hale presents the same solution to the concept *horse* paradox in his (2013a: \S 1.7–1.10). He further explains the idea that terms refer to properties only 'derivatively' in (Hale & Wright 2012: \S VIII).

(pp. 115–6), and Hale as rejecting Type-Kind Uniqueness whilst retaining Single Relation (pp. 116–7).

There is, I think, good reason to regret Hale and Wright's terminological change. 'The Reference Principle' has become a standard term in the literature on the concept horse paradox, and it is always used in Wright's 1998 sense.²³ But even more importantly, contra Hale and Wright's claim (2012: 105–6), Single Relation and Type-Kind Uniqueness do not jointly entail that singular terms cannot refer to properties. To see this, suppose that we accepted Single Relation and that we thought that singular terms can refer to properties. (The reader may find this instruction somewhat jarring: I have spent some time arguing that these suppositions are impossible to make! However, we are here trying to enter, as far as is possible, into Hale and Wright's discussion on its own terms.) We could still maintain that singular terms refer to and only to objects; it is just that some of those objects would also be properties. Similarly, we could still say that predicates refer to and only to properties; it is just that some of those properties would also be objects. And so on through the various different types of expression and kinds of thing. But this would be to say that there is a one-to-one correlation of the sort described by Type-Kind Uniqueness. What is true is that Single Relation and Type-Kind Uniqueness will entail that singular terms cannot refer to properties if we build in the assumption that the different kinds of thing are mutually exclusive. However, Hale and Wright are not free to make this assumption, as they portray Wright as accepting Type-Kind Uniqueness despite the fact that he treats properties as a subclass of objects.

In light of this confusion, I think that we will be better off if we return to Wright's 1998 use of 'the Reference Principle' and the original characterisations of Hale and Wright's responses to the concept *horse* paradox. Now the question is: How successful are those responses?

Let us start with Hale. The first thing to note is that Hale's redefinitions of 'object' and 'property' obviously cannot play any role in blocking the concept *horse* paradox: you cannot define yourself out of a problem. In reality, the working part of Hale's response is simply his rejection of the Reference Principle. His redefinitions merely allow him to reject that principle without giving up on the Fregean slogan 'No property is an object'. (I am not quite sure why Hale (e.g. 2012: 119) is so keen to keep hold of that slogan, given that it has come to mean something quite different in his hands, but I will not pursue that matter here.) Now, Hale is of course free to use 'property' and 'object' however he likes, but so am I. Earlier I said that objects are those things which can be referred to by singular terms. That was before we had introduced Hale's distinction between primary and derivative reference. So I must now be quite clear: I am taking objects to be those

 $^{^{23}}$ See, for example, (Oliver 2005: 177), (Noonan 2006: 167), (Dolby 2009: 286), (Textor 2010: 130), (MacBride 2011: 299) and (Trueman 2012: 98).

things which can be referred to primarily or otherwise by singular terms;²⁴ similarly, I am taking properties to be those things which can be referred to primarily or otherwise by predicates. As far as I can tell, these clarifications make no difference to my earlier argument. If it worked the first time around then it will still manage to show that it is nonsense to say that some property is an object. Moreover, in my terminology if not his own, Hale *is* trying to say that some property is an object. So, assuming my argument to be successful, we can see that Hale's response to the concept *horse* paradox is unworkable.

We turn now to Wright. We have already seen how Wright's distinction between reference and ascription allows him to avoid the concept *horse* paradox when it is presented as a consequence of the Reference Principle. But can it do the same work when we present the concept *horse* paradox as I did in $\S\S4-6$? The answer is, I believe: No. At no stage in my argument did I assume that predicates refer to properties in the same way that terms refer to objects. We could re-run the argument of \S 4–6 with all talk of predicates referring to properties replaced with talk of predicates ascribing properties. The only point at which the change from reference to ascription could possibly make any real difference is in §5, where I insisted that if we are to count the predicate ' ξ is a horse' as a referring expression then there must be an analogue of (4) for that predicate. But even here, I do not think the argument will suffer. So long as ascription is supposed to be the analogue of reference for predicates, then we will require some principle which is to ' ξ is a horse' and ascription as (4) is to 'Julius Caesar' and reference. Moreover, it is a straightforward consequence of my own argument that it is nonsensical to suppose that singular terms and predicates refer in the same way. In §5 I tried to show that we must take the analogue of ' ξ refers to ζ ' for predicates to be of the form ' ξ refers to ϕ '. And if, as I have argued, it is nonsense to say that a property is an object, then it is also nonsense to say that ξ refers to ζ ' and ' ξ refers to ϕ ' are co-extensive, let alone that they co-refer. (The difference between my ' ξ refers to ϕ ' and Wright's ' ξ ascribes ζ ' is that his is a *first-level* predicate, as Wright (2012: 118–9) makes clear, and mine is a *mixed-level* predicate.)

So, neither Hale's nor Wright's responses to the concept *horse* paradox are successful; we still cannot make sense of referring to properties with singular terms. Perhaps things would have been different if the paradox really were the child of the Reference Principle, but as we have seen, it is not. This is not to say that there is no relation between the paradox as I understand it and the Reference Principle. Clearly, if my argument is successful it can be generalised into an argument for the following claim:

If two expressions α and β are nowhere intersubstitutable (at the level of sense) then it is nonsense to suppose that α refers (in whatever sense expressions of α 's type refer) to what β refers (in whatever sense expressions of β 's type refer) to.

But crucially, this principle has at no point been *assumed*; it has been *argued for*.

²⁴Perhaps we could amicably agree to call Hale's objects 'bobjects' and my ones 'robjects'.

8. CONCLUSION

In this paper I have argued that it is nonsense to say that a property is an object. How interesting is this conclusion? Well, one might think that it would not be all that interesting if we simply denied that there are any properties. I will not examine this nominalist line of response here, but I will make one remark. Almost all of the existing debate over whether properties exist has been carried out under the assumption that if they do exist then we can refer to them with singular terms. It is not, therefore, clear what we should make of this debate now that we have seen that this assumption is mistaken.

For now, then, let us continue to assume that there are properties. Can we make this assumption and at the same time live with the concept *horse* paradox? Hale and Wright (2012: §III) argue that we cannot. One of their main arguments (pp. 97–8 & 103–4) is that if we cannot refer to properties with singular terms, then we do not have the means to express a semantics which involves assigning referents to predicates. However, this is a little premature. In §§5–6 I argued that we must find second-level analogues for the predicates ' ξ refers to ζ ', ' ξ is an object' and ' $\xi = \zeta$ '; that is, second-level predicates which are to properties as these three predicates are to objects. If we do manage to find them then we will be able to use them to say things like 'Predicates refer to properties', 'The predicate " ξ is a horse" does not co-refer with the predicate " ξ is a dog" ', and so on after all.

Nonetheless, there are things that we will not be able to say. If we cannot make sense of identifying a property with an object, then we cannot say of an object what we can say of a property. First-level predicates are true or false of objects, and so if we cannot say that a property is an object then we cannot say that first-level predicates are true or false of properties; similarly, second-level predicates are true or false of properties, and so if we cannot say that a property is an object then we cannot say that second-level predicates are true or false of objects.²⁵ Objects and properties have become in a sense incomparable. The best we can do is say something of objects, take a breath, and say something else of properties. What we are left with is an inviolable quietism about the property/object distinction.

This conclusion is not as extreme as it might sound. In this paper I have nowhere argued that we cannot use abstract singular terms like 'horsiness' or even 'the property *horse*', although using the latter invites confusion. For all I have said, then, we can talk about horsiness and say things like 'for any x, x is a horse iff x instantiates horsiness'. My point is only that we cannot say of the referent of the *term* 'horsiness' what we can say of the referent of the *predicate* ' ξ is a horse'.

Still, the quietism I have argued for can be hard to stomach. Matters are only exacerbated by the fact that in the course of presenting my own argument I appear to

²⁵See (Frege 1892: 189; 1891–5: 175), (Geach 1976: 57–8) and (Dummett 1981a: 177–8).

have frequently said of properties what can be said of objects. Indeed, even to say something like 'it is impossible to say about properties what we can say about objects' is to try to make just the kind of comparison between properties and objects which I have argued we cannot make! This is a precarious dialectical position, but I do not think that it is ultimately unstable.²⁶ It is impossible for me to do full justice to this problem here, but I would like to give a sketch of what I take to be the correct response. At the start it seemed to Benno that he could say of properties what he can say of objects. We have tried to show him by his own lights, not ours, that he cannot. We did this the only way we could: by speaking in ways that Benno was happy with. But we are not thereby committed to the meaningfulness of this dialogue with Benno. All we require is that it seem to Benno, who started out convinced that we were talking meaningfully, that he is presented with an argument which eventually shows him that he cannot say of a property what he can say of an object. Now, the ultimate point of this encounter with Benno is not to convince him to make a new comparison between properties and objects, namely that they are in a certain sense incomparable. Rather, we want Benno to give up on trying to compare properties and objects altogether.²⁷

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 $^{^{26}}$ My predicament is exactly the same as the one in which Wittgenstein found himself at the end of the the *Tractatus* (6.54). This should come as no surprise. As Geach (1976) convincingly argues, Wittgenstein's saying/showing distinction has its roots in his reflections on the concept *horse* paradox.

 $^{^{27}}$ See (Button 2010: §IV) for a similar line of thought.

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