The Philosophy of Physics
Lecture Four

Minkowski Spacetime

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Minkowski Spacetime

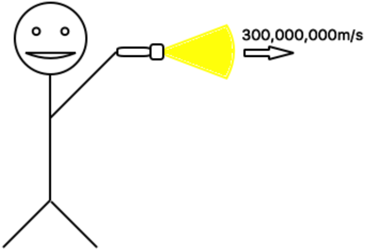
The Two Postulates of SR

(1) **The Relativity Postulate**: the laws of nature are the same in all inertial frames of reference

(2) **The Light Postulate**: the speed of light (in a vacuum) is a constant: c

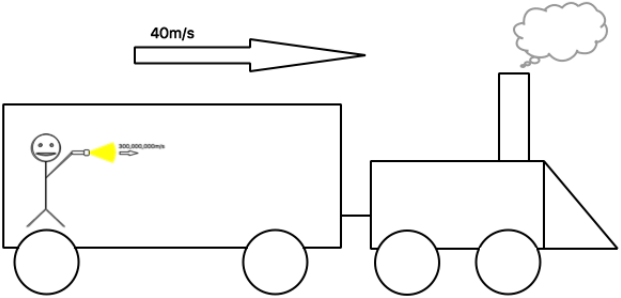
- These two postulates initially look mutually inconsistent

An Apparent Inconsistency



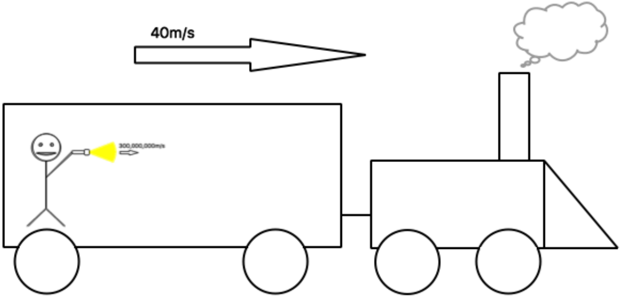
- Imagine you stand on a train, shine a torch and measure the speed of the light coming from the torch as c

An Apparent Inconsistency



- From the position of the station, we would normally expect to measure the speed of light as $c + 40m/s$

An Apparent Inconsistency

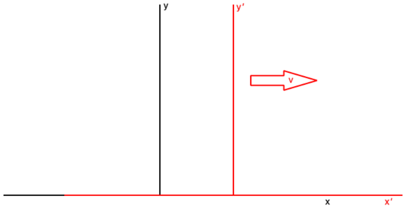


- But given the Light and Relativity postulates, we must measure it as c!

Galilean Transformations

- This looks contradictory, but that is only because we have a background assumption about how to add velocities

$$t' = t$$
$$y' = y$$
$$z' = z$$
$$x' = x - vt$$



- These equations are known as **Galilean Transformations**

Lorentz Transformations

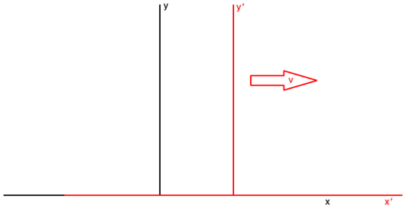
- We need to replace these Galilean transformations with the following equations:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$



- These equations are known as **Lorentz Transformations**

A Curiosity...

- SR is meant to be a theory of mechanics, i.e. a *general* theory about how things move through spacetime
- But one of the *basic postulates* of SR mentions how light *in particular* moves through spacetime
- It turns out that you can re-express SR as a geometry of spacetime

The Spacetime of Relativity

- The Lorentz Transformations characterise a particular kind of spacetime, called **Minkowski spacetime**
- As we will see, the geometry of Minkowski spacetime is **not** Euclidean
- However, the geometry of Minkowski spacetime is also quite different from the particular non-Euclidean geometries we looked at in Lecture 2

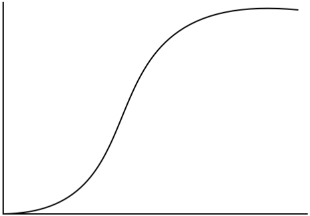
Minkowski Spacetime

Different Levels of Geometrical Structure

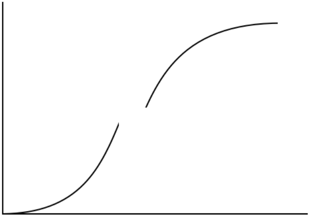
- We can distinguish between different levels of geometrical structure:
  - Topological Structure
  - Affine Structure
  - Metrical Structure

The Topological Structure

- The **topology** of a geometry tells us which collections of points form **continuous paths** in that geometry



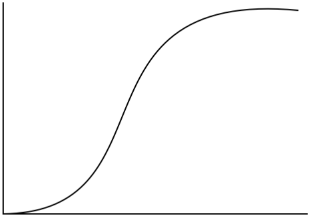
continuous



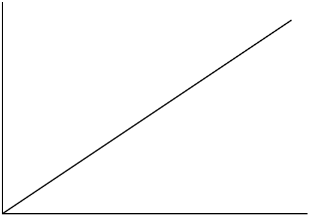
discontinuous

The Affine Structure

- The **affine structure** of a geometry tells us which continuous paths form **straight lines** in that geometry



curved



straight

The Metrical Structure

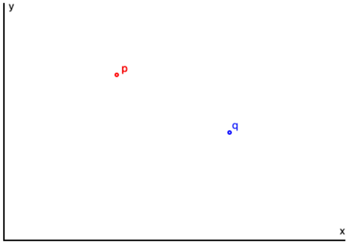
- The **metric** of a geometry tells us how far apart a given pair of points is in the geometry
- The metric might say that the distance between point p and point q is 5
- 5 what? 5 metres? 5 centimetres? 5 lightyears? None of these! The metric doesn't come with units!
- The metric just tells us the **ratios** between distances
  - If the distance between p and q is 5, but the distance between r and s is 1, then the distance between p and q is 5 times the distance between r and s

The Metrical Structure

- All of the spacetimes that we are going to look at in this lecture have exactly the same topology and affine structure
  - They all agree on what counts as a continuous path, and which continuous paths count as straight lines
- But they all disagree over the **metric**
  - They all disagree about how far apart points in the spacetime are

The Metrical Structure of Euclidean Geometry

- The most familiar metrical structure is that of Euclidean geometry
- In 2-dimensional Euclidean geometry, we can use **Pythagoras' Theorem** to figure out the distance between any two points



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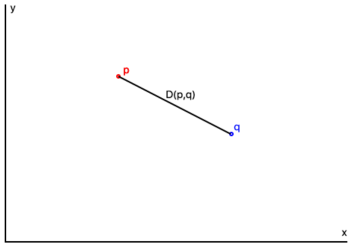

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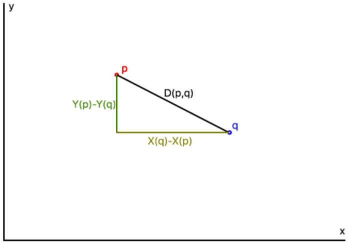


- $D(p,q) = \sqrt{(X(p) - X(q))^2 + (Y(p) - Y(q))^2}$

The Metrical Structure of Euclidean Geometry

- You can use the same method to determine the distance between two points in Euclidean geometry, *no matter how many dimensions you are working with*
- **3 dimensions:**
  - $D(p,q) = \sqrt{(X(p)-X(q))^2 + (Y(p)-Y(q))^2 + (Z(p)-Z(q))^2}$
- **4 dimensions:**
  - $D(p,q) = \sqrt{(X(p)-X(q))^2 + (Y(p)-Y(q))^2 + (Z(p)-Z(q))^2 + (T(p)-T(q))^2}$

Minkowski Spacetime

From Space...

- We are used to thinking of Space as a three-dimensional continuum of spatial points
- We can represent each spatial point with three numbers, (x, y, z)
- Each spatial point is the potential location of some body

...To Spacetime

- **Spacetime** is a **four**-dimensional continuum of points
- We can represent each spacetime point with four numbers, (x, y, z, t)
- Each spacetime point is the potential location of some **event**

Newtonian and Galilean Spacetime

- **Newtonian spacetime** incorporates three principles:
  (1) Spacetime can be divided up into absolute *simultaneity hyperplanes*
  (2) There is a definite spatial distance between any two points on a given simultaneity hyperplane
  (3) There is a definite spatial distance between any two points on *different* simultaneity hyperplanes
- **Galilean spacetime** includes (1) and (2), but not (3)

Differences in Metrical Structure

- Newtonian and Galilean spacetimes both have the topology and affine structure of 4-dimensional Euclidean geometry
  - They agree on what counts as a continuous path, and which continuous paths count as straight lines
- But they disagree over the metrical structure

The Newtonian Metric

- Newtonian spacetime has exactly the same metrical structure as 4-dimensional Euclidean geometry

$$D(p,q) = \sqrt{(X(p) - X(q))^2 + (Y(p) - Y(q))^2 + (Z(p) - Z(q))^2 + (T(p) - T(q))^2}$$

- This isn't the **spatial** distance between p and q, but the **overall** spatio-temporal distance

The Galilean Metric

- The metric of Galilean spacetime is Euclidean **within a given simultaneity hyperplane**

$$D(p,q) = \sqrt{(X(p) - X(q))^2 + (Y(p) - Y(q))^2 + (Z(p) - Z(q))^2}$$

- But this metric does not define a distance between different points on different simultaneity hyperplanes

- It does define a **temporal** distance between points on different hyperplanes, but not a **spatial** distance
  - $X(p) - X(q)$ is undefined if p and q are on different hyperplanes

- As a result, it is impossible to compute an overall spatio-temporal distance

Minkowski Spacetime

Introducing Minkowski Spacetime

- Minkowski spacetime also has a Euclidean topology and affine structure
- And like Newtonian spacetime, the metric is defined over every pair of points
- But it has an **importantly** different definition

$$I(p,q) = \sqrt{(T(p)-T(q))^2-(X(p)-X(q))^2-(Y(p)-Y(q))^2-(Z(p)-Z(q))^2}$$

- The difference between this Minkowski **interval** and the Euclidean **distance** is that the Time parameter has a different sign to all the others
  - Time is positive, the others are all negative

Minkowski Spacetime is the Spacetime of SR

- What is the relationship between SR and Minkowski spacetime?
- The Minkowski interval is **invariant** under Lorentz transformations
  - Suppose that the interval between p and q is x according to inertial frame of reference A
  - We then use the Lorentz transformations to move over to another inertial frame of reference B
  - We will still find that the interval between p and q is x according to B
- Although the interval is invariant, spatial and temporal distances **are not**
  - Different frames of reference factor the interval into spatial and temporal distances in different ways

Empirical Content

- How can we tell whether our spacetime is a Minkowski spacetime?

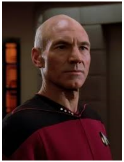
- We need to add some hypotheses about how measurable phenomena relate to the spacetime
  (1) **The Law of Light:** The interval between any two points on the spacetime path of a beam of light is 0
  (2) **The Clock Hypothesis:** The amount of time that a clock shows to have elapsed between two events is proportional to the interval along the clock's spacetime path between those two events

- With these two assumptions in place, a world with a Minkowski spacetime would exhibit exactly the relativistic phenomena discussed last week

Minkowski Spacetime

The Twins Paradox

- On the day of their 40th birthday, Picard flies off in the Enterprise



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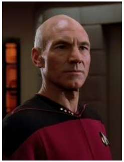
The Twins Paradox

- On the day of their 40th birthday, Picard flies off in the Enterprise
- He travels at 0.66c until the day of his 43rd birthday, and turns around



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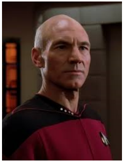
The Twins Paradox

- On the day of their 40th birthday, Picard flies off in the Enterprise
- He travels at 0.66c until the day of his 43rd birthday, and turns around
- He gets back to Earth on the day of his 46th birthday



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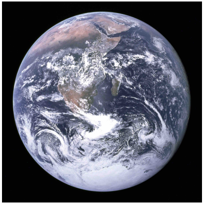
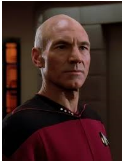


The Twins Paradox

- On the day of their 40th Birthday, Picard flies off in the Enterprise
- He travels at 0.66c until the day of his 43rd birthday, and turns around
- He gets back to Earth on the day of his 46th birthday
- But he discovers Kirk preparing to celebrate his 48th birthday!



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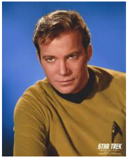

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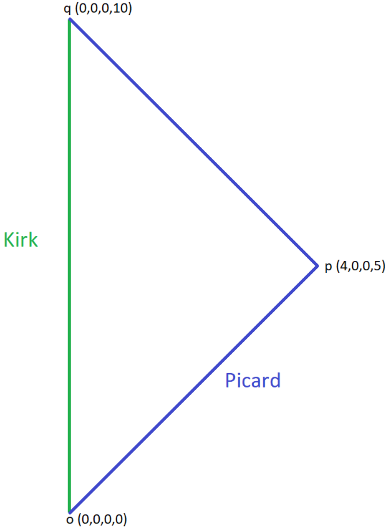
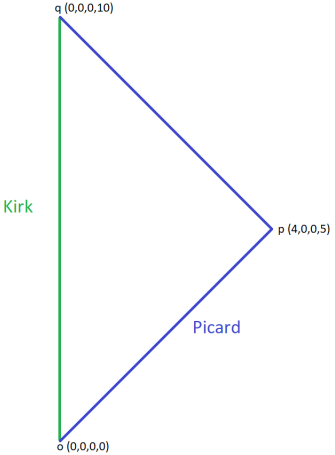
The Twins Paradox

- From Kirk's point of view, Picard moved away really fast, which is why time moved more slowly for Picard than for Kirk
- But isn't it also true that from **Picard**'s point of view, **Kirk** moved away really fast?
- So why didn't time move more slowly for Kirk?
  - If Kirk was moving at 0.66c relative to Picard, then a 4.5 years should have passed for Kirk during Picard's 6 year space trip
  - So Kirk should only have been 44 and a half when Picard got back, not nearly 48!

A Minkowski Diagram

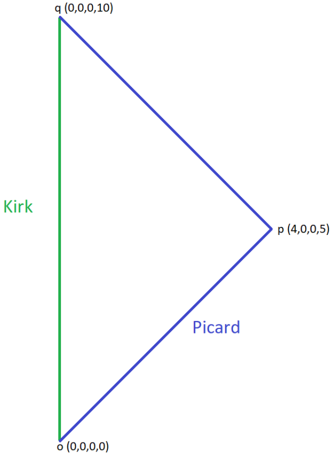


A Minkowski Diagram



- $I(o,q) = \sqrt{10^2 - 0} = 10$
- $I(o,p) = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3$
- $I(p,q) = 3$
- $I(o,p) + I(p,q) = 6$

A Minkowski Diagram

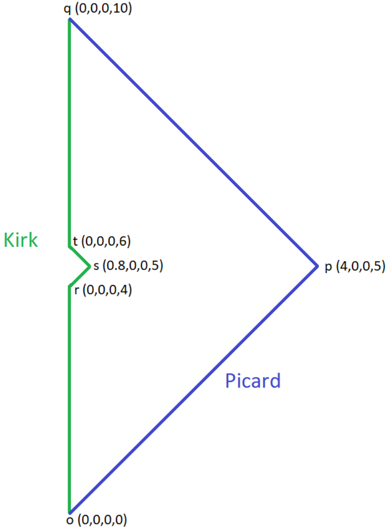


- Kirk's path is longer than Picard's in Minkowski spacetime, and so by the Clock Hypothesis, he'll measure more time passing
- Of course, Kirk's path **looks** shorter than Picard's in this diagram
- But that is just because we are representing a non-Euclidean spacetime in a (near-enough) Euclidean medium

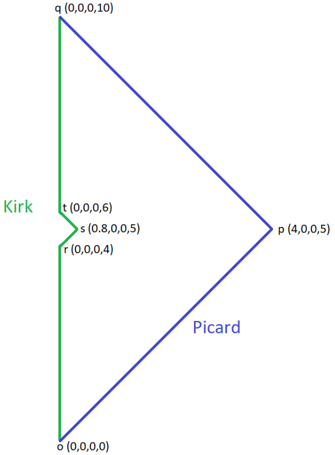
Acceleration

- When we first discussed the Twins Paradox, we mentioned the common idea that the symmetry between Picard and Kirk is broken by the fact that it is Picard who accelerates, not Kirk
  - We know it is Picard who accelerates, because he is the one who feels various forces acting on him
- However, we also saw that that answer wasn't quite right: we can set up a more complex triplets paradox in which no triplet is accelerating
- And we can see what is wrong with the "acceleration"-answer even more clearly now that we can drawn Minkowski diagrams

Another Minkowski Diagram

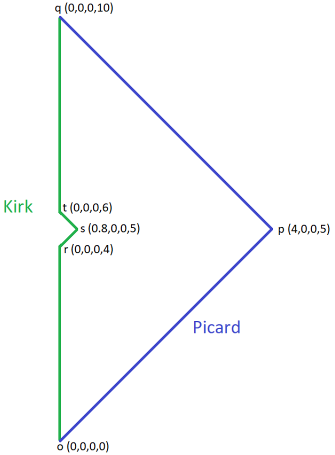


Another Minkowski Diagram



- Now Kirk is accelerating just as much as Picard
- But Pcard's spacetime path is **still** longer than Kirk's

Another Minkowski Diagram



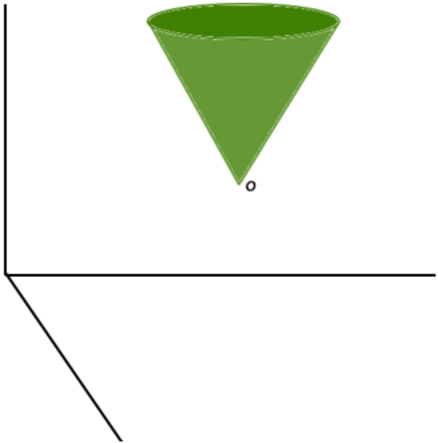
- $I(o, r) = \sqrt{4^2 - 0} = 4$
- $I(r, s) = \sqrt{1^2 - 0.8^2} = 0.6$
- $I(o, r) + I(r, s) + I(s, t) + I(t, q) = 4 + 0.6 + 0.6 + 4 = 9.2$
- $I(o, p) + I(p, q) = 6$

Minkowski Spacetime

Future Light Cones

- Imagine a spacetime point, o, at which a light is turned on
- The light that was emitted from o will spread out in all directions, and so will form the surface of a sphere
- We can't really represent this in a two dimensional spacetime diagram, but if we suppress one of the spatial dimensions we will get a series of ever expanding circles, which together form a cone
- This is called the **future light cone** of o

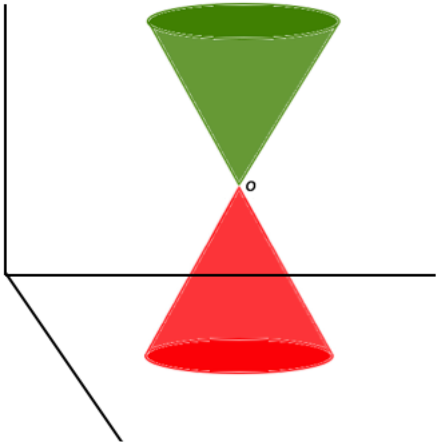
Future Light Cones



Past Light Cones

- We can also represent all of the light that is received at o with a cone
- This cone will expand backwards in time
- This is called the **past light cone** of o

Past Light Cones



Absolute Light Cone Structure

- We can draw light cones like this for every point in the spacetime
- Importantly, this light cone structure is **absolute**, meaning every frame of reference agrees on it
  - By the Law of Light, the interval between points on a beam of light's spacetime path is always 0
  - The interval is invariant between frames of reference
  - So every frame of reference agrees on the possible paths light beams can take from/to a given spacetime point

Time-like Separation

- If an event lies in either the past or future light cone of o, we say that it is **time-like** separated from o

- Every observer will agree that an event in the past light cone of o is temporally earlier than o
  - Such an event is in o's **absolute past**

    (However, different observers will disagree about just how far back in o's absolute past this event is)

- Every observer will agree that an event in the future light cone of o is temporally later than o
  - Such an event is in o's **absolute future**

    (However, different observers will disagree about just how far ahead in o's absolute future this event is)

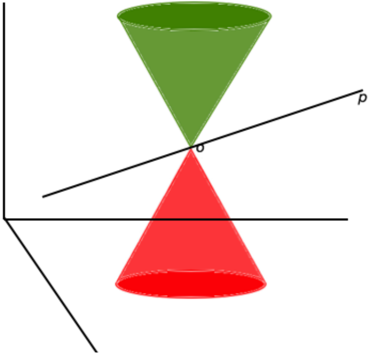
Light-like Separation

- If an event lies on the surface of one of o's light cones, we say that it is **light-like** separated from o
- If two events are light-like separated, then the interval between them is 0!
- This just goes to show how different the Minkowski interval is from the ordinary Euclidean distance

Space-like Separation

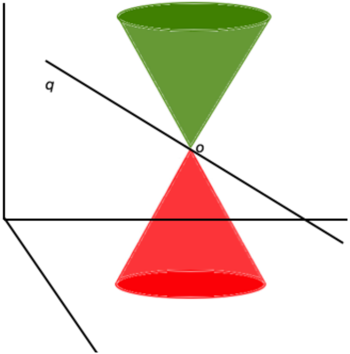
- If an event lies outside of o's light cones, we say that it is **space-like** separated from o
- If o and p are space-like separated, then there is some frame of reference according to which they are simultaneous
- According to that frame of reference, o and p are separated in space, but not time
- But bear in mind, this goes for o and **any** event that it is space-like separated from
  - Any point which cannot reach/be reached from o by a signal travelling no faster than light is space-like separated from o
  - For each such point, p, there is a frame of reference according to which o is simultaneous with p

Relativity of Simultaneity



- Simultaneity is always relative to a frame of reference in SR!

Relativity of Simultaneity



- Simultaneity is always relative to a frame of reference in SR!

Minkowski Spacetime

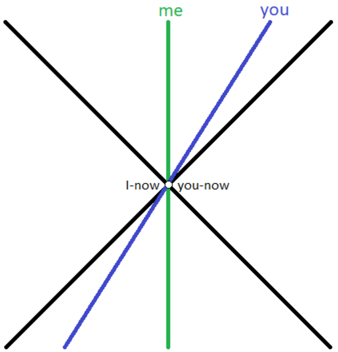
Dynamic Conceptions of Time

- We ordinarily think of time in dynamic terms: time “flows” or “passes”
- This way of speaking can naturally lead to **presentism**, the philosophical view that only the things which exist **now** are real
  - Things which exist only in the future will be real, but aren’t yet
  - Things which exist only in the past were real, but aren’t any more
- Or slightly less extremely, it can lead to the **growing-block** view
  - Things which exist only in the past are real, just like things which exist in the present
  - But things which exist only in the future still aren’t real

SR and the Static Conception of Time

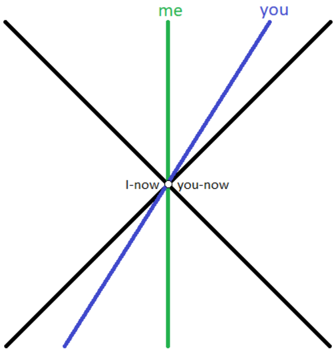
- But lots of philosophers think that these dynamic conceptions of time are **refuted** by SR
  - There is no such thing as **the** present
  - Something is in the present if it is simultaneous with us right now
  - But different events will count as simultaneous with us right now relative to different frames
- Instead, we should adopt the **static conception** of time, according to which everything in spacetime is equally as real as everything else

Putnam's Argument



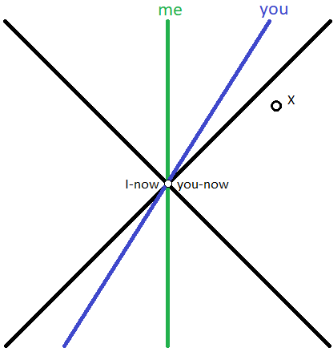
- Imagine that we are moving very fast relative to each other, but cross paths at a certain point

Putnam's Argument



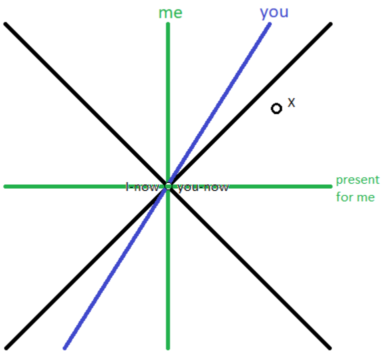
- I certainly agree that you are real, since we occupy the same spacetime point

Putnam's Argument



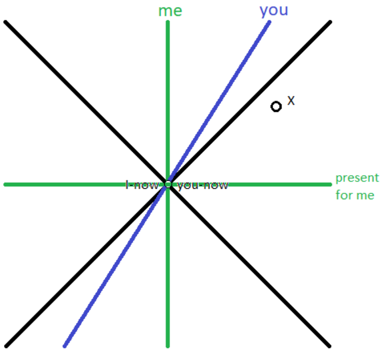
- Now consider some point X which is space-like separated from us

Putnam's Argument



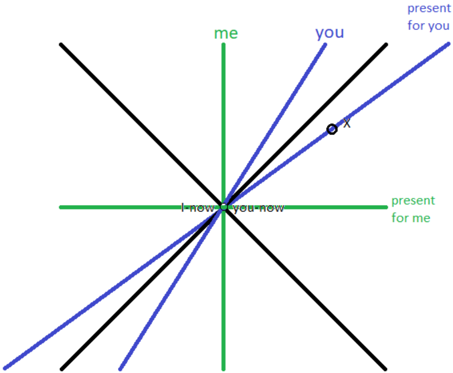
- And suppose that in my frame of reference, X is in the future (i.e. X comes after I-now)

Putnam's Argument



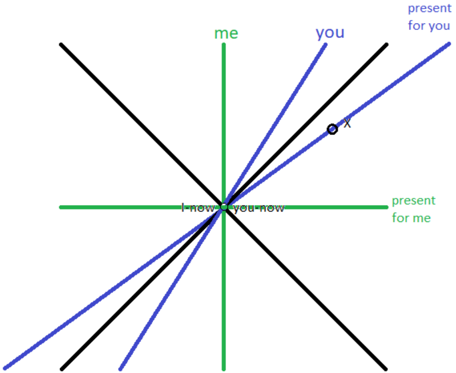
- If I am a presentist, I will want to say that X is not real, since it is in my future

Putnam's Argument



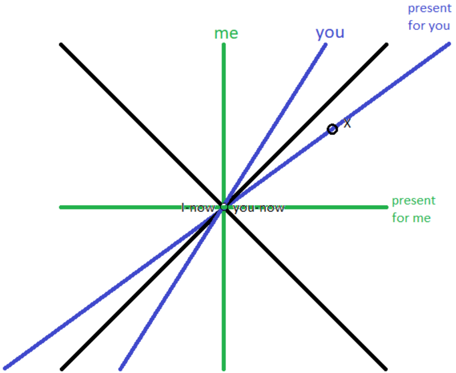
- But nonetheless, it may be that in your frame of reference, X is simultaneous with you-now and I-now

Putnam's Argument



- So you will want to say that X is real, since X is in the present moment

Putnam's Argument



- But surely, if you-now are real for me-now, and X is real for you-now, then X must be real for me-now!

Putnam's Argument

- Putnam's argument thus uses the relativity of simultaneity in SR to argue that (at least some) events in your future are already real
- This argument makes a structural assumption:
  - If you are real for me, and X is real for you, then X is real for me
- This seems like a **very** plausible assumption
- Maybe a presentist could reject it, but it would require a substantial re-working of our ordinary conception of reality

Putnam's Argument

- Alternatively, a presentists could block the argument by insisting that there is a **privileged** inertial frame of reference
  - Two events are *really* simultaneous iff they are simultaneous according to the privileged frame of reference

- Does this amount to rejecting SR? Tricky!
  - SR tells us that the laws of physics are the same in in all inertial frames of reference; in the sense, no inertial frame is *physically* privileged
  - It is strictly consistent with this to say that some inertial frame is *metaphysically* privileged
  - But still, many philosophers think that it goes against the *spirit* of SR to privilege any inertial frame for any purpose

The Static Conception of Time

- If we don't want to modify SR or mess with our conception of reality, then we seem to have little choice but to accept the static conception of time
  - The past, present and future are all as real as each other
- It can be hard to think in these static terms, and it can even be a little scary
  - We've all suffered horrible events, and take comfort from them being over
  - But on the static picture, those bad events still exist
- However, we can also try to take comfort from the picture...

Maybe It's Not All Bad

- Einstein sent this letter to the family of his friend Michele Besso after Besso died:

now he has preceded me a little by parting from this strange world. This means nothing. To us believing physicists the distinction between past, present and future has only the significance of a stubborn illusion.

(Quoted in Foslin, Albert Einstein, A Biography, p. 741)

Seminar Reading

- For the seminar, please read:
  - Kristie Miller, ‘Presentism, Eternalism and the Growing Block’
  - Hilary Putnam, ‘Time and Physical Geometry’
  - Katherine Brading, ‘Presentism as an Empirical Hypothesis’
- All of these are available via the Reading List on the VLE