The Philosophy of Physics
Lecture Three

Introduction to Special Relativity

Rob Trueman
rob.trueman@york.ac.uk

University of York

Introduction to Special Relativity

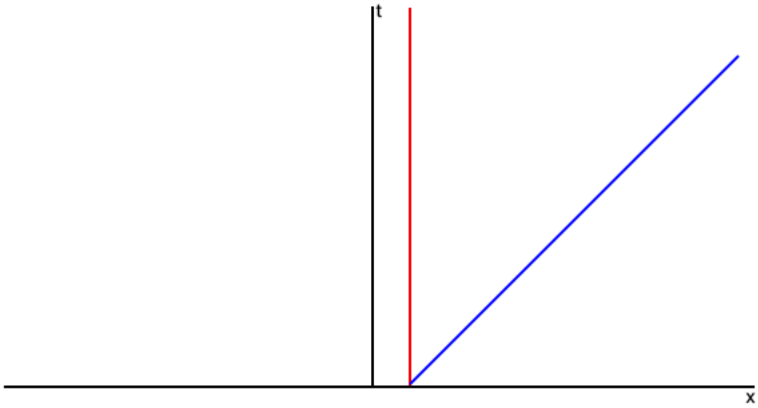
The First Postulate

(1) **The Relativity Postulate**: the laws of nature are the same in all inertial frames of reference

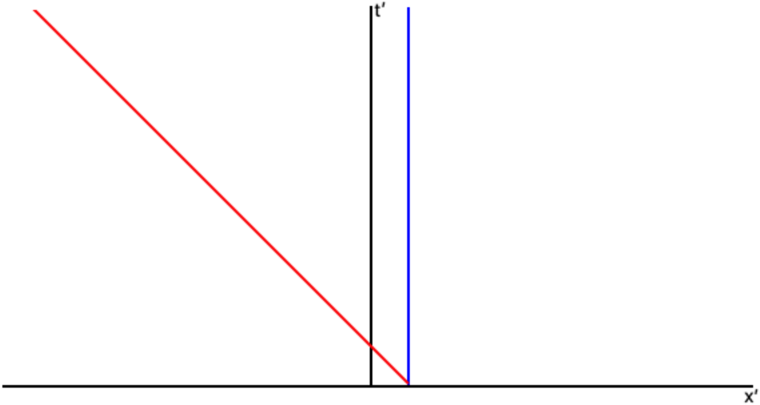
Reminder: Frames of Reference

- A frame of reference is a system of co-ordinates we can use to describe where everything is in time and space
- There are lots of different frames of reference
- Some frames of reference are "moving" relative to other frames
  - All this means is that some of the bodies that are at rest according to one frame of reference are moving according to another

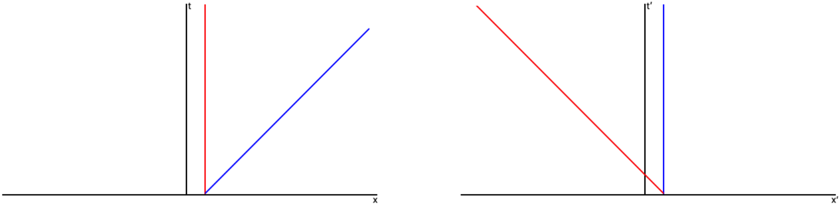
Reminder: Frames of Reference



Reminder: Frames of Reference



Reminder: Frames of Reference



Frame A Frame B

- Relative to A, B is moving to the right on the x-axis
- Relative to B, A is moving to the left on the x-axis

Reminder: Frames of Reference

- An **inertial** frame is a frame which is not accelerating (i.e. is moving at a constant speed in a straight line)
- In other words: no accelerating body is represented as either being at rest or having a constant velocity by an inertial frame
  - If a body is at rest or has a constant velocity, then we call its path through spacetime an **inertial path**, and it is represented by a straight line
- Remember that there is an objective fact of the matter about whether a body is accelerating or not
  - Accelerating objects are affected by measurable forces

Back to the First Postulate

(1) **The Relativity Postulate**: the laws of nature are the same in all inertial frames of reference

- This postulate tells us that no laws of nature single out a particular inertial frame as special
- There is no physical way of telling whether an inertial frame of reference is **really** at rest or **really** moving with a constant velocity
- A stronger version of relativity: **there is no such thing** as "really" being at rest or "really" moving with a constant velocity

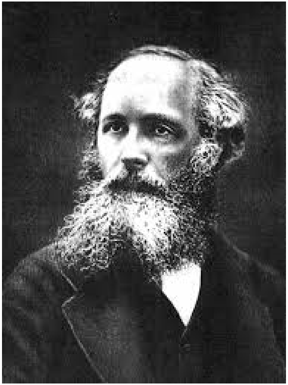
The Second Postulate

(2) **The Light Postulate**: the speed of light in a vacuum is a constant: c

The Origins of the Second Postulate

- In 1860, Maxwell realised that electric and magnetic fields could interact in a way that produces self-propagating electromagnetic waves

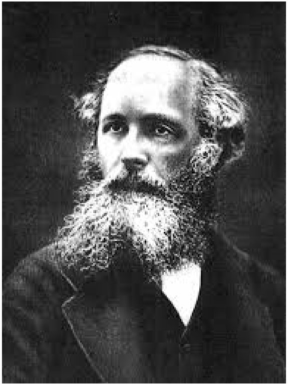
- He was also able to calculate the speed that these waves would move at approximately $3 \times 10^8 m/s$ (or just c for short)



James Clerk Maxwell

The Origins of the Second Postulate

- Scientists had already discovered experimentally that light moves at this speed
- So Maxwell conjectured that light was an electromagnetic wave
- We now know that Maxwell was right



James Clerk Maxwell

The Origins of the Second Postulate

- Normal waves (like water waves and sound waves) are waves **in** a medium
- So if light is a wave, then it is natural to think that it is a wave **in** a medium too, which was called the luminiferous aether
- We could then explain what we mean when we say that light waves have a certain velocity
- Light waves travel at $3 \times 10^8 m/s$ *relative to* the aether

The Origins of the Second Postulate

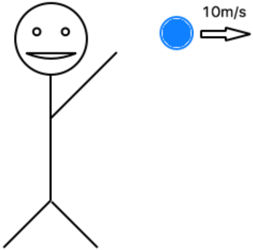
- If light was a wave in the aether, then the speed at which we measure light moving should be affected by the speed **we** are moving relative to the aether
- However, in a famous series of experiments, Michelson and Morley demonstrated that we always measure light as having the same speed, no matter what direction we move in (at a constant velocity)
- This simultaneously proved that there was no aether, and gave us the second postulate of SR
  - More on all this in Lecture 5!

The Two Postulates

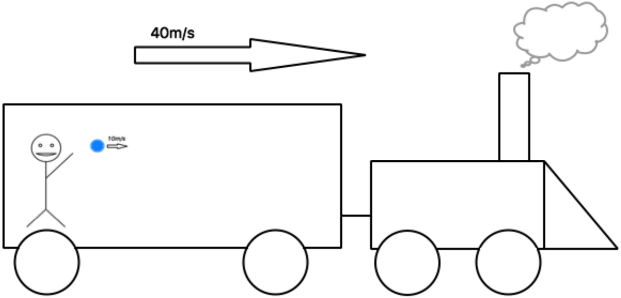
(1) **The Relativity Postulate**: the laws of nature are the same in all inertial frames of reference

(2) **The Light Postulate**: the speed of light (in a vacuum) is a constant: c

Galilean Addition of Velocities



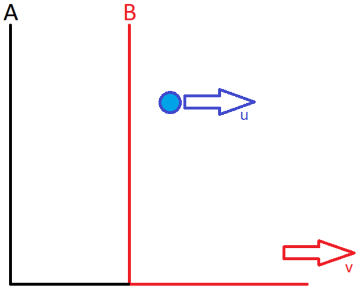
Galilean Addition of Velocities



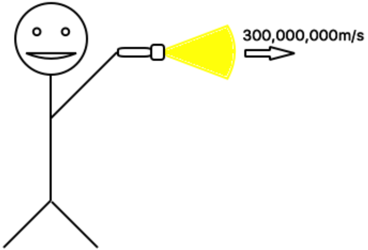
- From the position of the station, we measure the speed of the ball as being $10m/s + 40m/s = 50m/s$

Galilean Addition of Velocities

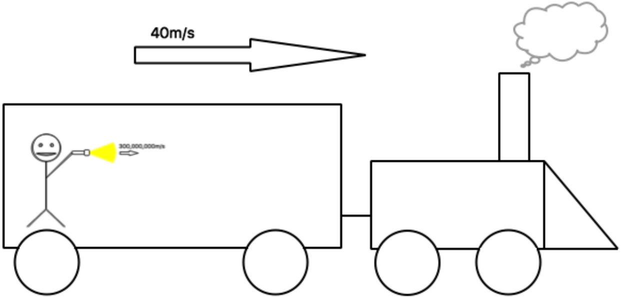
- Relative to Frame A, Frame B has velocity v on the x-axis
- Relative to Frame B, an object has u on the x-axis
- Relative to Frame A, that object has velocity $v + u$ on the x-axis



Galilean Addition of Velocities Fails

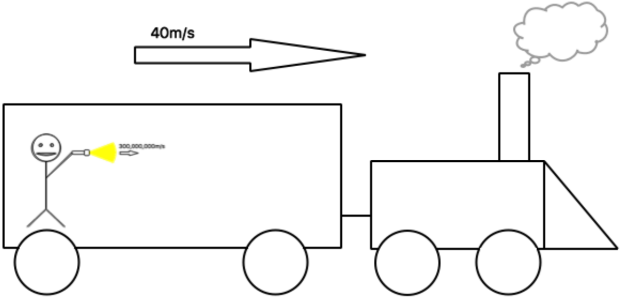


Galilean Addition of Velocities Fails



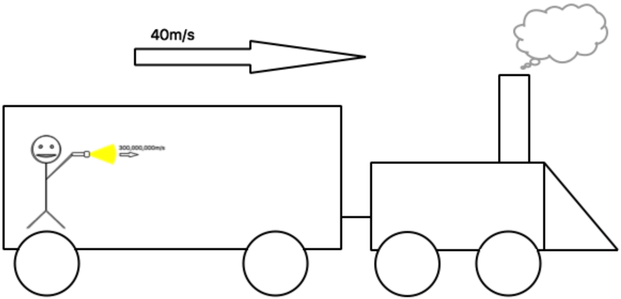
- From the position of the station, we would expect to measure the light on the train as being $c + 40m/s$

Galilean Addition of Velocities Fails



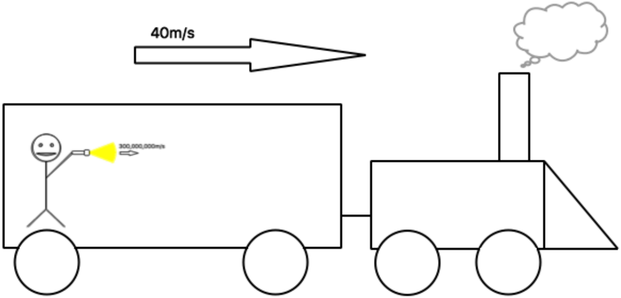
- But the Light Postulate is a law of physics saying that the speed of light is always c

Galilean Addition of Velocities Fails



- And the Relativity Postulate tells us that the laws of physics are the same in **all** inertial frames of reference

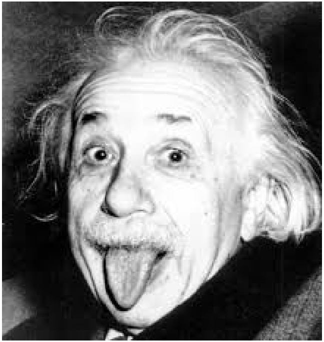
Galilean Addition of Velocities Fails



- So from the position of the station, we measure the speed of the light on the train as c, not $c + 40m/s$!

Enter Einstein

- The First and Second Postulates of Relativity look inconsistent, but Einstein showed us how to make them work together
- We need to give up on the Galilean ideas about how to translate from one inertial frame of reference to another
- Special Relativity (SR) tells us how we should make these translations

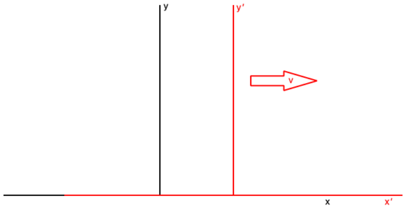


Albert Einstein

Introduction to Special Relativity

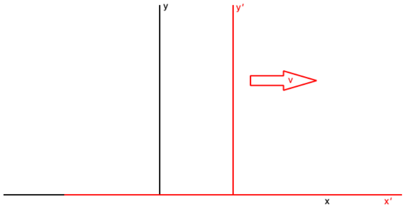
Galilean Transformations

- $t' = t$
- $y' = y$
- $z' = z$
- $x' = x - vt$



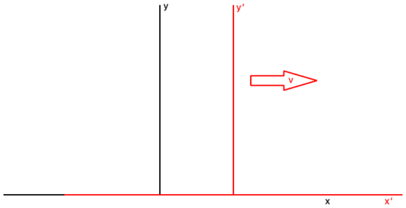
Galilean Transformations

- $t' = t$



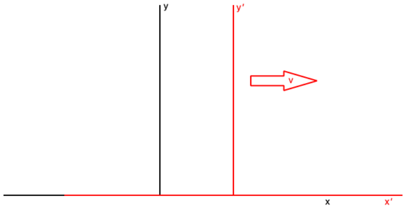
Non-Galilean Transformations

- $t' \neq t$



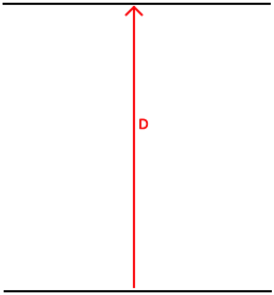
Non-Galilean Transformations

- $\Delta t' \neq \Delta t$



A Light Clock

A Light Clock



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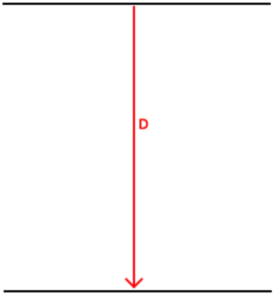

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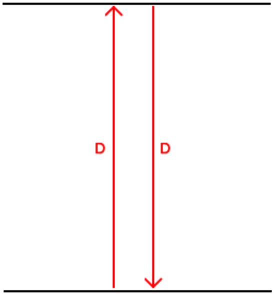

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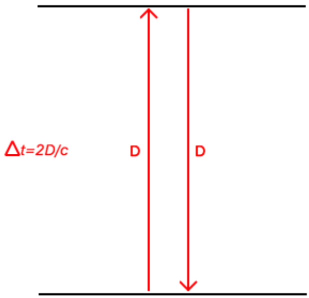
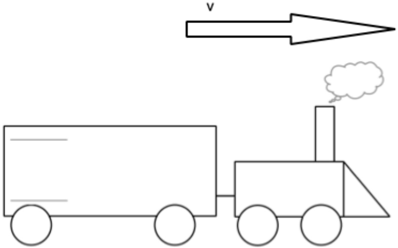
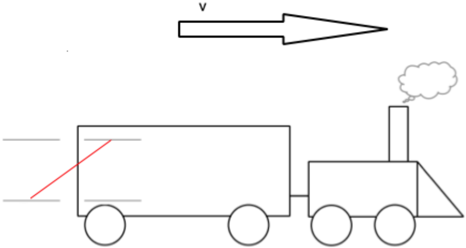


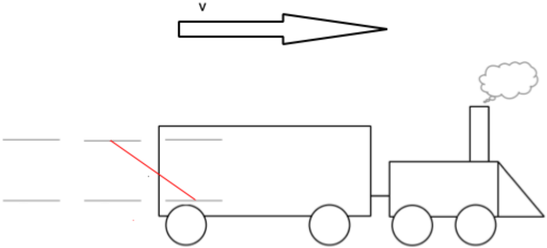
A Light Clock on a Train



A Light Clock on a Train



A Light Clock on a Train



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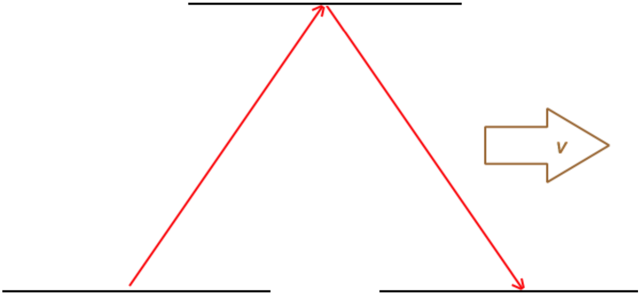

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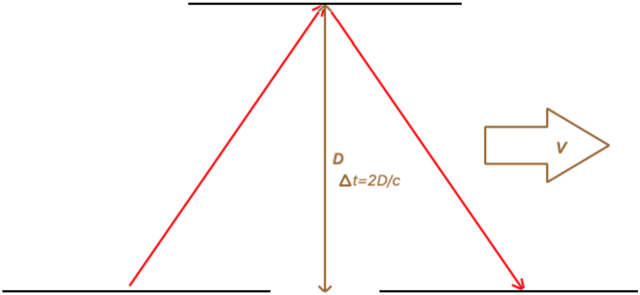

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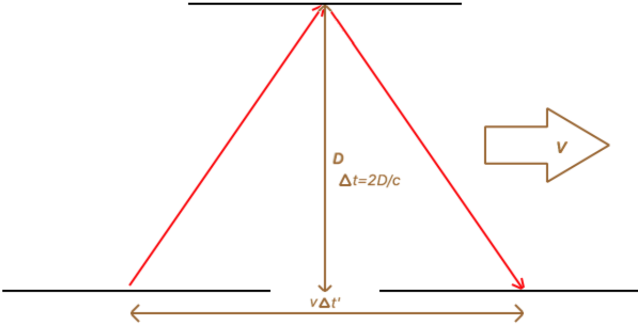

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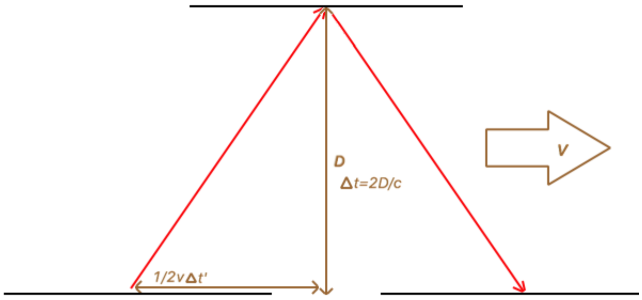

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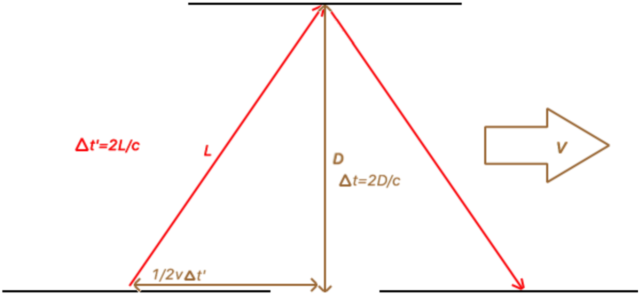


Calculating Time Dilation

$$L = \sqrt{(\tfrac{1}{2} v \Delta t')^2 + D^2}$$

$$\Delta t' = \frac{2L}{c}$$

$$\Delta t' = \frac{2\sqrt{(\frac{1}{2} v \Delta t')^2 + D^2}}{c}$$

$$\Delta t' = \frac{\sqrt{(v \Delta t')^2 + (2D)^2}}{c}$$

$$(\Delta t')^2 = \frac{(v \Delta t')^2 + (2D)^2}{c^2}$$

$$(\Delta t')^2 = \frac{v^2}{c^2}(\Delta t')^2 + (\frac{2D}{c})^2$$

$$(\Delta t')^2 - \frac{v^2}{c^2}(\Delta t')^2 = (\frac{2D}{c})^2$$

$$(\Delta t')^2(1 - \frac{v^2}{c^2}) = (\frac{2D}{c})^2$$

$$(\Delta t')^2 = \frac{(\frac{2D}{c})^2}{1 - \frac{v^2}{c^2}}$$

$$\Delta t' = \frac{\frac{2D}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \frac{2D}{c}$$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

An Example

- Suppose that a train is moving at 0.8c relative to a train station
- From the point of view of the people on the station, how much time has to pass for the clock on the train to read 100s?

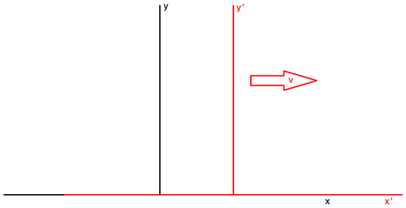
An Example

- $\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$
- $\Delta t = 100s$
- $v = 0.8c$
- $\Delta t' = \frac{100s}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}}$
- $\Delta t' = \frac{100s}{\sqrt{1 - \frac{0.64 \times c^2}{c^2}}}$
- $\Delta t' = \frac{100s}{\sqrt{1 - 0.64}}$
- $\Delta t' = \frac{100s}{\sqrt{0.36}}$
- $\Delta t' = \frac{100s}{0.6}$
- $\Delta t' = 166.67s$

Moving clocks run slow!

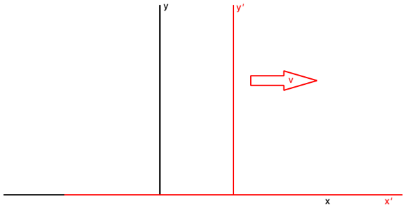
Galilean Transformations

- $t' = t$
- $y' = y$
- $z' = z$
- $x' = x - vt$



Lorentz Transformations

- $t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$
- $y' = y$
- $z' = z$
- $x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$



Introduction to Special Relativity

From the Point of View of those at the Station

- If a train is moving at 0.8c relative to a train station, then from the point of view of someone at the station, it takes 166.67 seconds for a clock on the train to read 100s
- From the point of view of the people at the station, everything going on in the train is happening in slow motion
- **But how do things on the station look from the view point of the train?**

From the Point of View of those on the Train

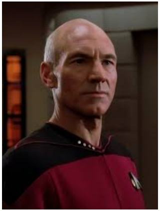
- From the viewpoint of someone on the train, it is the station that is moving at 0.8c
- So from the point of view of someone on the train, it takes 166.67 seconds for a clock on the station to read 100s
- From the point of view of the people in the train, everything going on at the station is happening in slow motion

Symmetry Broken

- At this point, you might think that time dilation effects are always symmetric
- But the **Twins Paradox** shows that this is not true
  - It isn't *really* a paradox, just a surprising (and empirically verified!) consequence of SR

The Twins Paradox

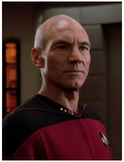
James T Kirk



Jean-Luc Picard

The Twins Paradox

- On the day of their 30th birthday, Picard flies off in the Enterprise



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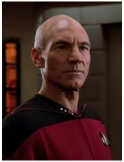
The Twins Paradox

- On the day of their 30th birthday, Picard flies off in the Enterprise
- He travels at 0.66c until the day of his 33rd birthday, and turns around



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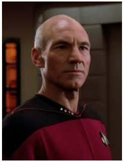
The Twins Paradox

- On the day of their 30th birthday, Picard flies off in the Enterprise
- He travels at 0.66c until the day of his 33rd birthday, and turns around
- He gets back to Earth on the day of his 36th birthday



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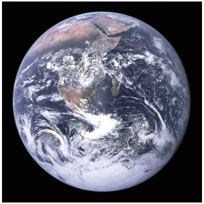
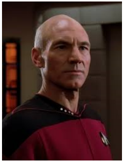


The Twins Paradox

- On the day of their 30th Birthday, Picard flies off in the Enterprise
- He travels at 0.66c until the day of his 33rd birthday, and turns around
- He gets back to Earth on the day of his 36th birthday
- But he discovers Kirk preparing to celebrate his 38th birthday!



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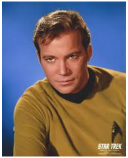

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The Twins Paradox

- From Kirk's point of view, Picard moved away really fast, which is why time moved more slowly for Picard than for Kirk
- But isn't it also true that from **Picard**'s point of view, **Kirk** moved away really fast?
- So why didn't time move more slowly for Kirk?
  - If Kirk was moving at 0.66c relative to Picard, then a 4.5 years should have passed for Kirk during Picard's 6 year space trip
  - So Kirk should only have been 34 and a half when Picard got back, not nearly 38!

Absolute Acceleration

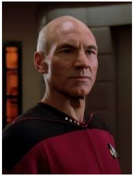
- **A common answer:** the Relativity Postulate tells us that we can't distinguish between inertial frames
- But Picard's frame **isn't** inertial!
  - He accelerates when he leaves Earth to get up to 0.66c
  - He then accelerates in the other direction to slow down, stop, and turn around
  - And then he accelerates one last time to come to a stop back at Earth
- Acceleration is absolute, so we know that it was Picard who really accelerated, and not Kirk
  - Picard feels lots of forces during his periods of acceleration, which Kirk does not feel

Absolute Acceleration

- This is a common answer, but it is wrong
- **We can set up the paradox without any accelerations!**

The Triplets Paradox

James T Kirk

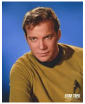


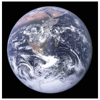
Jean-Luc Picard

Kathryn Janeway

The Triplets Paradox

- Picard flies past Kirk at 0.66c, and see that Kirk is celebrating his 30th birthday





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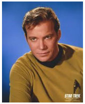
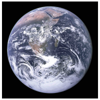
The Triplets Paradox

- Picard decides to set her own clock by this, and so celebrates his 30th birthday too





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The Triplets Paradox

- Picard keeps travelling, and on his 33rd birthday he crosses paths with Janeway, who is travelling at 0.66c in the opposite direction

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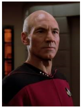
The Triplets Paradox

- Janeway sees that Picard is celebrating his 33rd birthday, and decides to set her own clock by this. So she celebrates her 33rd birthday too



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33

The Triplets Paradox

- Janeway keeps travelling, and on her 36th birthday she crosses paths with Kirk

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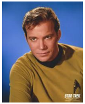

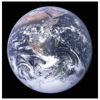

The Triplets Paradox

- But to Janeway's surprise, Kirk is preparing to celebrate his 38th birthday!

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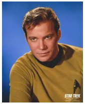

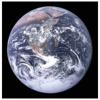

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The Triplets Paradox

- Kirk is travelling at 0.66c relative to Picard, and so from Picard's perspective, Kirk only ages 2.25 years in the 3 years it takes Picard to reach Janeway
- Similarly, Kirk is travelling at 0.66c relative to Janeway, so from Janeway's perspective, Kirk only ages 2.25 years in the 3 years it takes Janeway to reach him after crossing paths with Picard
- So Kirk should have only aged 4.5 years, but in reality he aged 8 years!
- **Where have the missing 3.5 years gone?**

What's Really Gone Wrong?

- The real issue that breaks the symmetry between Kirk and Picard/Janeway isn't to do with acceleration
- It is about the change in inertial frames on the Picard/Janeway path
- But to understand what is going on here, we need to understand what happens to the notion of **simultaneity** in SR

Introduction to Special Relativity

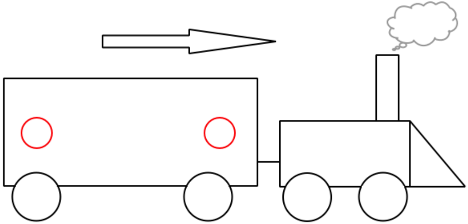
Common Sense

- Common sense tells us that there is an *absolute* fact of the matter about whether two events are simultaneous (i.e. happen at the same time)
- **More formally:** if two events are simultaneous according to *one* inertial frame of reference, then they are simultaneous according to *every* inertial frame of reference
- This bit of common sense is incorporated into classical, Newtonian mechanics
- But it is rejected by SR!

The Definition of Simultaneity

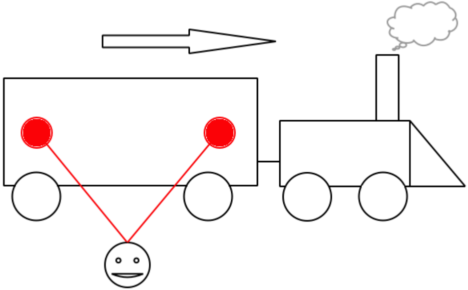
- Two spatially separated events, A and B, are simultaneous iff light rays sent off from A and B will meet a point midway between them, C, at the same moment
  - Since C is equidistant between A and B, a ray going from A to C travels the same distance as a ray of light going from B to C
  - Since the speed of light is constant, if a ray of light from A reaches C at exactly the same moment as a ray from B reaches C, then those rays must have been sent off at the same time
  - So A and B must be simultaneous
- It turns out that there are some interesting questions to ask about this definition, but we will save them for Lecture 5

The Relativity of Simultaneity



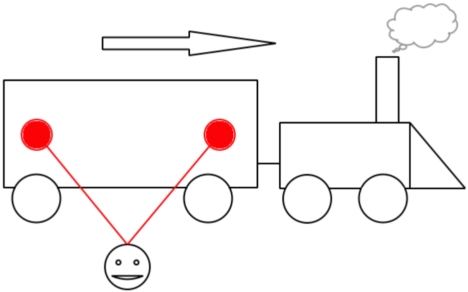
- There are two flash bulbs on a moving train

The Relativity of Simultaneity



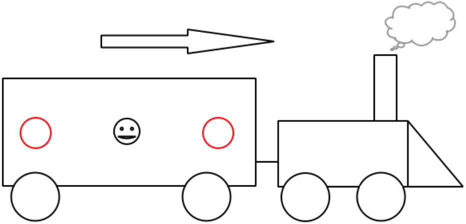
- They go off, and the rays reach an observer on the platform at the same time

The Relativity of Simultaneity



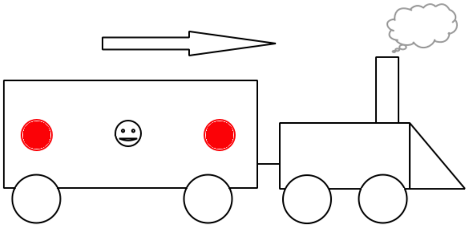
- The observer is equidistant between the two bulbs, and so from their perspective, they flashed at the same time

The Relativity of Simultaneity



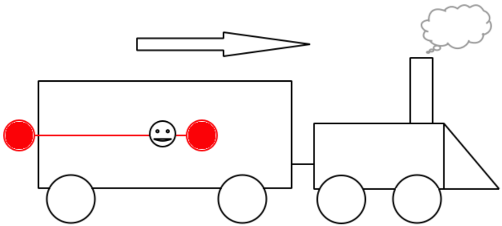
- But now imagine that there is someone inside the train

The Relativity of Simultaneity



- The bulbs go off, but the person in the train is moving towards the bulb on the right

The Relativity of Simultaneity



- The light from the bulb on the right will therefore reach the person in the train before the light from the bulb on the left

The Relativity of Simultaneity

- But now consider what the person on the train will see

The Relativity of Simultaneity

- Relative to this person's frame of reference, they remain constantly equidistant between the two bulbs

The Relativity of Simultaneity

- So they will say that the right bulb goes off **first**...

The Relativity of Simultaneity

- ...and the left bulb goes off **second**

The Relativity of Simultaneity

- So there is no **absolute** fact of the matter about whether two events are simultaneous

- If two spatially separated events count as simultaneous according to **one** inertial frame of reference, then there will still be **another** inertial frame of reference according to which they occur at different times

Introduction to Special Relativity

What Went Wrong?

- Kirk is travelling at 0.66c relative to Picard, and so from Picard's perspective, Kirk only ages 2.25 years in the 3 years it takes Picard to reach Janeway
- Similarly, Kirk is travelling at 0.66c relative to Janeway, so from Janeway's perspective, Kirk only ages 2.25 years in the 3 years it takes Janeway to reach him after crossing paths with Picard
- So Kirk should have only aged 4.5 years, but in reality he aged 8 years!
- **Where have the missing 3.5 years gone?**

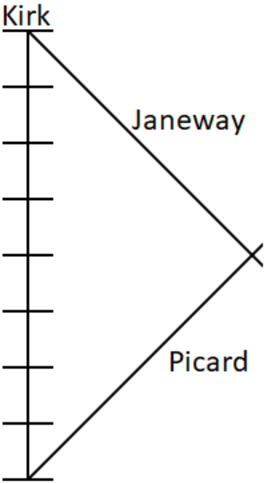
Simultaneity is not Absolute

- In the above reasoning, we are tacitly assuming that when Picard and Janeway cross paths, they will agree on how old Kirk is
  - We assumed we could figure out how much Kirk ages by adding how much he ages from Picard's perspective during Picard's journey to Janeway, and how much he ages from Janeway's perspective durin her journey to Kirk
  - That sum obviously won't work if Picard and Janeway don't agree on how old Kirk is when they cross paths
- But that is a mistake: **Simultaneity is relative to frames of reference!**

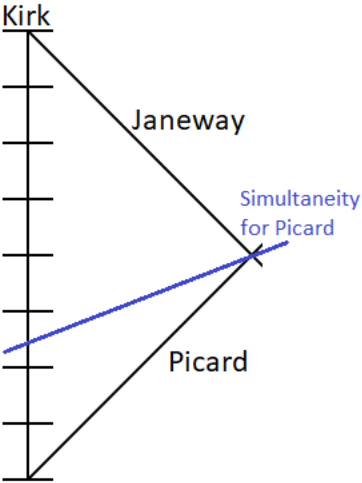
Simultaneity is not Absolute

- In Picard's frame of reference, his meeting with Janeway is simultaneous with Kirk's being 22.25
- But in Janeway's frame of reference, her meeting with Picard is simultaneous with Kirk's being 25.75
- That's where the missing 3.5 years are!

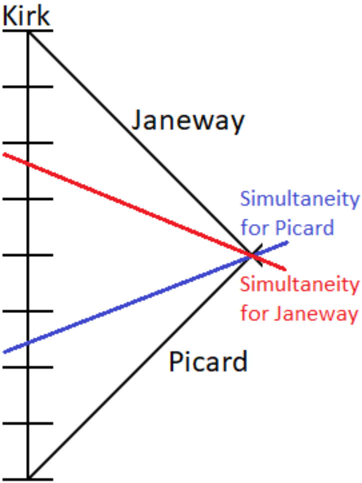
Simultaneity is not Absolute



Simultaneity is not Absolute



Simultaneity is not Absolute



For the Seminar

- In the seminar, we will look some more at SR, and see how to derive some of the other interesting consequences of the theory
- Make sure you read Part I of Einstein's *Relativity*