The Foundations of Mathematics
Lecture Seven

The Indispensability Argument

Rob Trueman
rob.trueman@york.ac.uk

University of York

The Indispensability Argument

Why Platonism?

- **Benacerraf's Dilemma** was meant to pose a general problem for every philosopher of mathematics
  - If taken at **face-value**, mathematical truths appear to imply **platonism**
  - But the **Access Problem** appears to push us toward **nominalism**

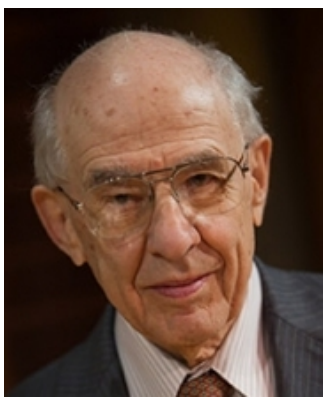
- However, the philosophical community seems to be more impressed by the Access Problem than the semantic argument for platonism
  - Perhaps because we could just try *denying* that mathematical claims are true (when taken at face value)?

- So why does anyone think we should be platonists?!

The Quine-Putnam Indispensability Argument

- The **Indispensability Argument** is the most influential contemporary argument for platonism
- Versions of this argument were first presented by Quine, and further developed by Putnam
- It is designed to convince *scientific realists* that they should believe in mathematical entities for just the same reasons that they believe in theoretical entities

WVO Quine



Hilary Putnam

The Indispensability Argument

(1) We ought to believe in all of the entities that are indispensable to our best scientific theories

(2) Mathematical entities are indispensable to our best scientific theories

$\therefore$ (3) We ought to believe in mathematical entities

In Putnam's Words...

if one is a realist about the physical world, then one wants to say that the Law of Universal Gravitation makes an objective statement about bodies — not just about sense data or meter readings. What is the statement? It is just that bodies behave in such a way that the quotient of two numbers associated with the bodies is equal to a third number associated with the bodies. But how can such a statement have any objective content at all if numbers and 'associations' (i.e. functions) are alike mere fictions?

In Putnam's Words...

It is like trying to maintain that God does not exist and angels do not exist while maintaining at the very same time that it is an objective fact that God has put an angel in charge of each star and the angels in charge of each of a pair of binary stars were always created at the same time! If talk of numbers and 'associations' between masses, etc. and numbers is 'theology' (in the pejorative sense), then the Law of Universal Gravitation is likewise theology.

(Hilary Putnam, 'What is Mathematical Truth' in his Philosophical Papers vol. 1, pp. 74–5)

The Indispensability Argument

Motivating Premise (1)

- Why should we believe Premise (1) of the **Indispensability Argument**?

  (1) We ought to believe in all of the entities that are indispensable to our best scientific theories

- This premise is usually motivated by combining two principles
  - **Naturalism:** We should accept our best scientific theories, and believe in all of the entities that they posit
  - **Confirmational holism:** Statements cannot be confirmed in isolation; when a theory is confirmed, the *whole* theory is confirmed altogether

- We'll look at naturalism now, and confirmational holism in the next section

Weak Naturalism vs. Strong Naturalism

- The version of naturalism needed for the first premise is actually fairly weak
  - **(Weak) Naturalism:** We should accept our best scientific theories, and believe in all of the entities that they posit
  - **Strong Naturalism:** We should accept our best scientific theories, believe in all of the entities that they posit, *and believe in nothing else*
- Strong naturalism is popular these days (among philosophers and the wider community)
- However, it is unclear to me exactly why it is so popular, and we don't need it for the Indispensability Argument

The Appeal of Scientific Anti-Realism

- Our naturalism might now sound so weak that you can't imagine anyone denying it, but plenty of philosophers have

- Modern physics posits all sorts of strange unobservable entities
- **Example: electrons**
  - Electrons are point-sized particles, but they're also waves
  - Electrons can become entangled with each other, so that changing the sate of one electron can lead to an instantaneous change in the state of another electron on the other side of the universe

An Example of Anti-Realism: Instrumentalism

- According to **instrumentalism**, we should not accept our best scientific theories as *true*
- Rather, we should accept them as *useful instruments* for deriving statements about observable entities from statements about observable entities
- Instrumentalism about scientific theories is comparable to Hilbert's formalism about ideal mathematics
- **Big question:** If our best scientific theories aren't actually true, then why should they be *reliable* instruments for inferring truths about observable entities?

Another Challenge to Naturalism: First Philosophy

- Couldn't *all* of our evidence about the external world be radically deceptive?
  - Maybe I'm asleep, and my whole life has just been a dream?
  - Maybe we're all in the Matrix, and the real world is nothing like what we're experiencing?
  - Maybe there's an evil demon who enjoys misleading all of us about the real nature of the world?

- Descartes raised these kinds of sceptical worries in his *Meditations on First Philosophy*

- He thought he had a way of answering them, but he did still think that science needed philosophy to save it from scepticism

Rejecting First Philosophy

- According to Quine, "first philosophy" is badly misconceived
- Science doesn't rely on philosophy to secure it's foundation
- Science is our best (most successful) attempt to describe and understand the world
- So if we want to know how the world is, we can do no better than to ask scientists

Quine's Answer to Scepticism

- How do you know that the world is (mostly) as it appears to be?
- That's a question for **scientists**, not armchair **philosophers**
  - Scientists provide theories of perception, which explain how and why our experiences accurately reflect how things are
- This obviously won't satisfy anyone who wants an external justification for science as a whole
- But Quine thinks that all we can ask for is an internal, scientific justification of how we practise science

In Quine's Words...

I philosophize from the vantage point of our own provincial conceptual scheme and scientific epoch, true; but I know no better.

(Quine, 'Speaking of objects', p.25)

Piecemeal Change

- Of course, Quine isn't saying that we mustn't ever revise our best scientific theories
- But crucially, we can only revise it piece-by-piece
- If we are wondering whether to revise the scientific claim P, our considerations will presuppose a vast body of background scientific knowledge
- There is no vantage point from which we can ask whether or not to accept science as a whole

In Quine's Words...

The naturalistic philosopher begins his reasoning within the inherited world theory as a going concern. He tentatively believes all of it, but believes also that some unidentified portions are wrong. He tries to improve, clarify, and understand the system from within. He is the busy sailor adrift on Neurath's boat.

(Quine, 'Five milestones of empiricism', p.72)

Against Instrumentalism

- Instrumentalists don't want to accept Quantum Mechanics as *true*, because the unobservable entities it deals with (e.g. electrons) are too problematic
- But that is to impose a **philosophical** standard of 'problematic'
- **Scientists** are happy to accept Quantum Mechanics as true
- So, since there is no external standard by which to judge scientific practice, we shouldn't be instrumentalists

The Indispensability Argument

Should You Believe in Electrons?

- According to **naturalism** you should accept Quantum Mechanics as true, and believe in all of the entities it posits
- But what exactly is involved in accepting QM as true?
- And which entities does QM posit?
- For example, does accepting QM as true automatically require believing in electrons?

Reformed Instrumentalism?

- Imagine a **reformed instrumentalist**, who accepts Quine's naturalism, but still denies that electrons exist
- This reformed instrumentalist *says* they accept QM as true, but they don't think that this means they have to accept *all* of the theorems of QM as true
- They just have to accept all of the *empirically confirmed* theorems of QM as true
- Likewise, they think that QM only posits a certain kind of entity if one of its *empirically confirmed* theorems says that that kind of entity exists

Confirmational Holism

- This kind of reformed instrumentalism is blocked by Quine's **confirmational holism**
  - Statements cannot be confirmed in isolation; when a theory is confirmed, the *whole* theory is confirmed altogether
- The reformed instrumentalist thinks that our evidence for QM really just confirms *some* of QM's theorems
- But according to confirmational holism, the evidence for QM confirms the theory *as a whole*

Confirming a Statement

- What evidence would confirm the statement 'Nick Offerman has a moustache'?
- *I've seen it!*
  - Maybe he's wearing a false moustache?
- *You could try to pull it off!*
  - Maybe he's super-glued on a false moustache?

- *You could do chemical experiments to show that it's his real hair!*
  - Maybe the chemical experiments are all invalidated by laboratory error?

Confirming a Theory

- At this point, some philosophers might worry that we can't really confirm that Nick Offerman has a moustache
- But instead, Quine just says that we cannot confirm that statement *in isolation*
- What we can do is confirm an entire *theory*, which includes that statement
  - This theory includes all sorts of background assumption, e.g. that Nick Offerman has not super-glued on a false moustache
- In just the same way, when a theory is *disconfirmed*, you could choose to revise any of the statements that make it up

Disconfirming a Theory

A physicist of the pre-Einsteinian era takes Newton's mechanics and his law of gravitation, N, the accepted initial conditions, I, and calculates, with their help, the path of a newly discovered small planet, p. But the planet deviates from the calculated path. Does our Newtonian physicist consider that the deviation was forbidden by Newton's theory and therefore that, once established, it refutes the theory N? No. He suggests that there must be a hitherto unknown planet p', which perturbs the path of p.

Disconfirming a Theory

He calculates the mass, orbit, etc. of this hypothetical planet and then asks an experimental astronomer to test his hypothesis. The planet p' is so small that even the biggest available telescopes cannot possibly observe it; the experimental astronomer applies for a research grant to build yet a bigger one. In three years time, the new telescope is ready. Were the unknown planet p' to be discovered, it would be hailed as new victory of Newtonian science. But it is not. Does our scientist abandon Newton's theory and his idea of the perturbing planet?

Disconfirming a Theory

No. He suggests that a cloud of cosmic dust hides the planet from us. He calculates the location and properties of this cloud and asks for a research grant to send up a satellite to test his calculations. Were the satellite's instruments (possibly new ones, based on a little-tested theory) to record the existence of the conjectural cloud, the result would be hailed as an outstanding victory for Newtonian science. But the cloud is not found. Does our scientist abandon Newton's theory, together with the idea of the perturbing planet and the idea of the cloud which hides it?

Disconfirming a Theory

No. He suggests that there is some magnetic field in that region of the universe which disturbs the instruments of the satellite. A new satellite is sent up. Were the magnetic field to be found, Newtonians would celebrate a sensational victory. But it is not. Is this regarded as a refutation of Newtonian science? No. Either yet another ingenious auxiliary hypothesis is proposed or . . . the whole story is buried in a dusty volume of periodicals and the whole story never mentioned again.

(Lakatos, 1970, 'Falsification and the methodology of scientific research programmes', pp.100–101)

Against Reformed Instrumentalism

- Reformed instrumentalists want to accept the confirmed statements in QM, and reject the rest
- But none of the individual statements of QM are confirmed in isolation
- They are all confirmed together
- If you are going to accept QM as true, you need to accept it *wholesale*

Motivating Premise (1)

- We can now see how Premise (1) of the **Indispensability Argument** is motivated
  - (1) We ought to believe in all of the entities that are indispensable to our best scientific theories
    - **Naturalism:** We should accept our best scientific theories, and believe in all of the entities that they posit
    - **Confirmational holism:** Statements cannot be confirmed in isolation; when a theory is confirmed, the *whole* theory is confirmed altogether

- **Question:** Where does *indispensability* come into it?

- **Answer:** If an entity isn't *indispensable* to a theory, then there's always the possibility that we could improve on our current best scientific theory by dispensing with it

The Indispensability Argument

Motivating Premise (2)

(2) Mathematical entities are indispensable to our best scientific theories

- Consider Newton's Law of Gravitation:

$$F = G\frac{M_a M_b}{d^2}$$

- We use numbers to represent the masses of a and b, the distance between a and b, and the force of attraction between them

- More complex physical theories refer to even more exotic mathematical entities, like metric tensors and vectors in Hilbert Spaces

Numbers and Unobservable Entities

- **The Indispensability Argument**
  - (1) We ought to believe in all of the entities that are indispensable to our best scientific theories
  - (2) Mathematical entities are indispensable to our best scientific theories
  - ∴ (3) We ought to believe in mathematical entities

- If this argument works, then we should believe in the existence of mathematical entities for *exactly the same reason* that we believe in unobservable physical entities

Three ways of Resisting the Argument

- **Option 1: Deny naturalism**
  - Our naturalism was so week, that this may look fairly implausible
- **Option 2: Deny confirmational holism**
  - Mary Leng attempts this in *Mathematics & Reality*
  - She claims that confirmational holism does not fit with the actual practice of scientists
- **Option 3: Deny that mathematics is indispensable to science**
  - Field attempts this as part of his *fictionalist* programme

The Indispensability Argument

Empirical Evidence for Mathematics

- If we accept the Quine-Putnam Indispensability Argument, can we use that to answer Benacerraf's epistemological **Access Problem**?
  - How can we possibly know anything about abstract mathematical entities?
- On the Quine-Putnam view, we know truths about mathematics *in exactly the same way* as we know any theoretical scientific claims:
  - We come up with scientific theories which then get confirmed as a whole
  - This gives us reason to accept all the sentences in the scientific theories, **including ones about mathemtical entities**

Problem 1: Mathematics is not A Priori

- We are used to thinking of mathematical truths as special: they can be known **a priori**
- But on the Quine-Putnam picture, they look as **a posteriori** as the rest of our scientific knowledge
- This was a result that Quine was more than happy with
  - According to Quine, the difference between mathematics and our empirical beliefs is just a matter of degree
  - We are reluctant to revise our mathematical beliefs because they are entrenched parts of so many theories, but we could revise our mathematical beliefs if we thought that was the best way of fixing a disconfirmed theory
- But although Quine was happy with this, we might not be!

Problem 2: A Missing Explanation

- How are we able to form beliefs about mathematical entities so reliably?
  - Why are we so good at introducing mathematical claims into our theory only when they accurately reflect the mathematical facts?
- We can explain (scientifically) why we are so reliable about (e.g.) electrons
  - We can detect the states of electrons with various experimental apparatus
  - We can offer theoretical explanations of how that apparatus work
- We seem unable to provide a similar explanation for mathematical entities (unless we side with Maddy...)

For the Seminar

- Required reading:
  - Mark Colyvan, *The Indispensability of Mathematics*, chs 1 & 2
- Study questions have been posted on the VLE
- Please remember to bring questions of your own!